REVISITING THE EFFECT OF SINE SWEEP RATE ON MODAL IDENTIFICATION

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NOMENCLATURE

a	acceleration amplitude
a_{max}	steady state peak amplitude
a_+ , a_-	swept peak amplitude
b	sweep rate ($R \ln(2) / 60$)
c	viscous damping
f	excitation frequency
f_0	natural frequency (Hz)
f_1	starting sweep frequency
f_{max}	steady state peak frequency
f_+ , f_-	swept peak frequency
k	stiffness
m	mass
R	sweep rate (oct/min)
t	time (s)
T_{ij}	dynamic transmissibility
$\widetilde{T}_{ij,0}$	effective transmissibility
u	displacement
η	non-dimensional sweep parameter
ω_0	natural frequency (rad/sec)
ζ_0	modal damping factor
ζ_+ , ζ_-	swept damping factor

ABSTRACT

Sine-sweep base excitation vibration tests are performed to qualify spacecraft structures in the low-frequency environment where the dynamic behavior can be characterized by a small number of modes. These modes can be extracted from the measured frequency response functions (FRF) by various modal identification methods, and then used for model validation purposes.

If the sweep rate is too high, the steady-state response is not reached and the resulting FRF profile will be modified by the presence of transients. This has a direct effect on the modal parameters extracted from the FRF.

Several authors have examined the effect of the sweep rate based on the response of a one-degree-of-freedom system. The goal of this paper is to adapt the approach to base-excitation vibration tests. Both increasing and decreasing exponential sweep rates are considered. The effect on the three principal modal parameters (natural frequency, damping and modal effective parameters) is examined.

1. INTRODUCTION

During sine-sweep vibration tests, if sweep rates are not sufficiently slow, the response will no longer be stationary (steady state) resulting in a modification of the response envelope due to transient behavior. This behavior has been studied extensively in the literature [1-5].

As an example, the response envelope at the resonance of a mode is shown in Figure 1 for positive and negative sweep rates, and compared to the steady state response.

We see that the shape and position of the resonant peak are altered as a function of the sweep rate and direction. Moreover a beat pattern or "ringing" following the peak may also appear. This ringing is a result of the system responding at two frequencies of nearly the same value comprising the free response at the natural frequency and the forced response at the swept excitation frequency.

In general, a frequency sweep will *decrease* the amplitude of the peak, *shift* the position of the peak (along the direction of the sweep), *broaden* the peak width and *distort* the shape of the peak. The higher the sweep rate, the more these effects are pronounced.

Consequently, the modal parameters identified from the response profile will also be perturbed. To quantify the influence of the sweep rate on the modal parameters, numerical simulation is performed using a base-excited 1-DOF system.

Details of the analysis are presented hereafter followed by a presentation of the results.

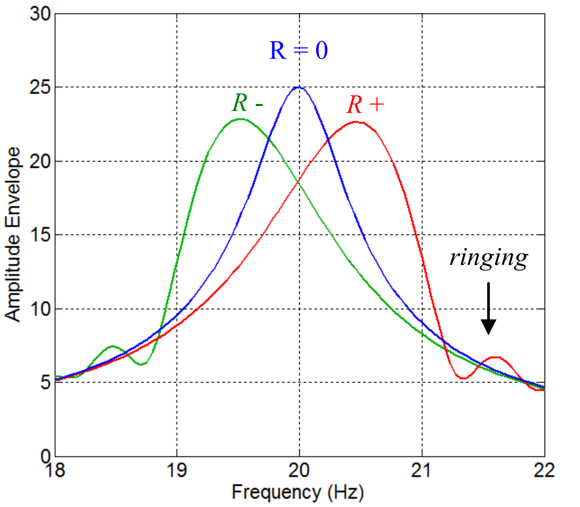
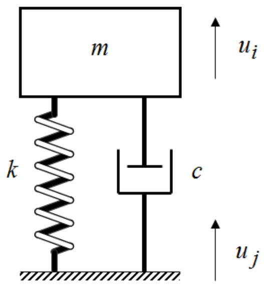


Fig. 1: Effect of Sweep Rate on Response Profile

2. ANALYSIS

2.1 The 1-DOF System

The 1-DOF system illustrated below with natural frequency f_0 (Hz) and viscous damping factor ζ_0 is excited at the junction *j* by an imposed acceleration $\ddot{u}_j$.



$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$$

$$\zeta_0 = c/2\sqrt{m\,k}$$

$$u_r = u_i - u_j$$

Fig. 1: 1-DOF System

The governing equation of motion expressed in terms of the relative motion u_r is given by:

$$\ddot{u}_r(t) + 2\,\zeta_0\,\omega_0\,\dot{u}_r(t) + \omega_0^2\,u_r(t) = -\ddot{u}_j(t) \qquad (1)$$

2.2 Harmonic Response

In the case of a harmonic excitation, Eq. (1) can be readily solved for $\ddot{u}_i$ in the frequency domain leading to the following expression for the dynamic transmissibility [6].

$$\frac{\ddot{u}_i(\omega)}{\ddot{u}_j(\omega)} = T_{ij}(\omega) = \frac{1 + i\,2\,\zeta_0\,\dfrac{\omega}{\omega_0}}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2 + i\,2\,\zeta_0\,\dfrac{\omega}{\omega_0}}\,\widetilde{T}_{ij,0} \qquad (2)$$

Note the dynamic transmissibility is a function of 3 modal parameters: f_0, ζ_0 and the effective transmissibility $\widetilde{T}_{ij,0}$. In the particular case of a 1-DOF system we have $\widetilde{T}_{ij,0} = 1$.

The amplitude of $T_{ij}(\omega)$ designated by $a(\omega) = |T_{ij}(\omega)|$ is plotted in Figure 2 and provides the envelope of an infinitely slow sine sweep response .Values for the peak frequency f_{max} and amplitude a_{max} derived from Eq. (2) are defined below [6].

$$f_{max} = f_0\,\frac{\sqrt{\sqrt{1 + 8\,\zeta_0^2} - 1}}{2\,\zeta_0} \qquad (3)$$

$$a_{max} = a(f_{max}) = \frac{1}{2\,\zeta_0}\sqrt{1 + 5\,\zeta_0^2 + \ldots} \qquad (4)$$

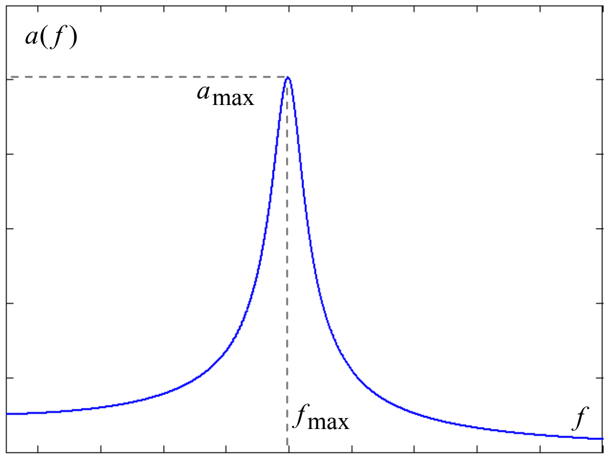


Fig. 2: Dynamic Transmissibility Amplitude

In the case of a sine sweep excitation, the response of the 1-DOF system is more or less perturbed depending on the sweep rate. This perturbation will have an effect on the peak amplitude a_{max} as well as on the modal parameters f_0, ζ_0 and $\widetilde{T}_{ij,0}$ identified from the perturbed response. The analysis and quantification of these effects are presented hereafter.

2.3 Sine Sweep Response

An exponential sweep rate is typically expressed in *R* octaves/minute such that after each minute the frequency f is multiplied by 2^R. This leads to the following expression for $f(t)$ with f_1 the frequency at the start of the sweep and $b = R\ln(2)/60$.

$$f(t) = f_1\, 2^{\frac{R}{60}t} = f_1\, e^{\frac{R\ln(2)}{60}t} = f_1\, e^{bt} \qquad (5)$$

The sine sweep excitation $\ddot{u}_j(t)$ of unit amplitude is derived from the frequency according to:

$$\ddot{u}_j(t) = \sin\left[2\pi \int_0^t f(\tau)\,d\tau\right] = \sin\left[2\pi \frac{f_1}{b}(e^{bt} - 1)\right] \qquad (6)$$

Note that Eq. (5) and (6) are valid for both positive and negative sweep rates by changing the sign of *b*.

Solving Eq. (1) for $\ddot{u}_i$ using the sine sweep excitation defined in Eq. (6) is performed by the use of recurrence formulas based on piecewise-linear interpolation of the excitation (see Craig [7] for example). A very small constant time step of $dt = 1/(200\, f_0)$ is used to accurately discretize the excitation and provide well-defined peaks in the response time history.

The characterization of the swept response is performed with the help of the amplitude envelope shown in Figure 3 and plotted versus the frequency $f(t)$.

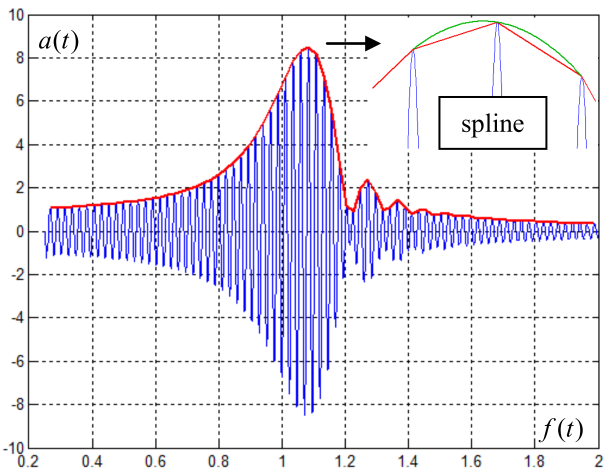


Fig. 3: Response Envelope for Characterization

A cubic spline interpolation is performed in the vicinity of the peak in order to obtain an improved estimation of the peak amplitude and frequency. Both upper and lower envelopes are considered – the one with the largest peak amplitude is used.

The following properties associated with the peak are identified from the envelope:

- a_+, a_- The peak amplitude for positive and negative sweep rates respectively.
- f_+, f_- The peak frequency for positive and negative sweep rates respectively.
- ζ_+, ζ_- The peak damping factor for positive and negative sweep rates respectively (obtained using the half-power method).

2.4 Sweep Parameter η

The non-dimensional sweep parameter, η, referred to in the literature allows deducing families of curves from single master curve expressed solely as a function of η.

The sweep parameter η is defined as the quality factor $Q_0 = 1/(2\zeta_0)$ divided by the number of cycles of excitation between the half-power points *N*.

$$\eta = \frac{Q_0}{N} \qquad (7)$$

For an exponential sweep rate, the number of cycles *N* is given by:

$$N = \frac{60\, f_0}{Q_0\, R\ln(2)} \qquad (8)$$

Introducing Eq. (8) into Eq. (7) leads to:

$$\eta = \frac{Q_0^2\, R\ln(2)}{60\, f_0} \qquad (9)$$

The assumption in using η is that all responses derived from parameters f_0, Q_0 and *R* with the same value of η will share the following characteristics:

- Fraction of maximum amplitude, a_+ / a_{max}, defined as the swept peak amplitude divided by the steady state peak amplitude.
- Ratio of damping, ζ_+ / ζ_0, defined as the swept damping factor divided by the steady state damping factor.
- Normalized frequency error, $\frac{f_+ - f_{max}}{f_{max}} Q_0$, defined as the relative shift in frequency multiplied by the quality factor.

For negative sweep rates the associated terms a_- / a_{max}, ζ_- / ζ_0 and $\frac{f_{max} - f_-}{f_{max}} Q_0$ are used.

As an example consider the two responses shown in Figure 4 calculated with different damping factors and sweep rates but with the same sweep parameter value $\eta \approx 4.62$.

The time histories are normalized with respect to a_{max} and plotted as a function of the frequency $f(t)$ to facilitate their comparison.

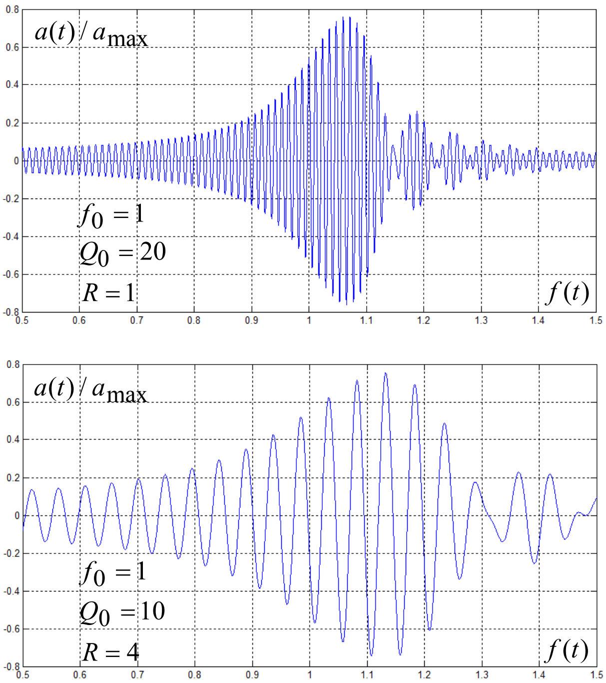


Fig. 4: Responses with Same Sweep Parameter η

Following analysis of the responses, we obtain the values shown below for both responses.

a_+ / a_{max}	ζ_+ / ζ_0	$\frac{f_+ - f_{max}}{f_{max}} Q_0$
0.76	2.0	1.3

For typical values of f_0, Q_0 and R, it is necessary to consider values of η in the range of $0.1 < \eta < 1000$. According to Eq. (9) this can be done by varying Q_0 and/or the term R/f_0 related to the sweep rate. The problem with varying R/f_0 is that the resolution (discretization) of the amplitude profile gets worse with increasing values of R/f_0 as witnessed in Figure 4. This becomes even more critical for large values of η. It is therefore better to vary Q_0 while maintaining a constant value for R/f_0.

3. RESULTS

3.1 Input Parameters

For this study a ratio of R/f_0=0.1 and values of Q_0 ranging from approximately 10 to 1000 were used to generate values of η from 0.1 to 1000 (see Table 1). This provides a very good resolution in the amplitude profile for all values of η while avoiding the use of excessively high damping factors.

Both positive and negative sweep rates were considered. To minimize the effect of initial transients, a starting frequency of $f_1 = f_0 / 4$ was chosen for positive sweeps, and $f_1 = 4 f_0$ for negative sweeps.

3.2 Master Curves

Responses were computed over the prescribed values of η and then analyzed to generate master curves for the changes in amplitude, damping and frequency as a function of η. These curves are plotted in Figures 5-7 and tabulated in Table 1.

All curves show a very smooth behavior over the entire range of values for η. This is a result of using a small value for R/f_0.

Moreover the curves are in excellent agreement with the results published by Hawkes [1] Cronin [2] and Lollock [3].

The master curves indicate nearly negligible differences between the positive and negative sweep rates. This was also observed by Cronin [2], but not by Lalanne [4] whose results show significant differences between positive and negative sweep rates. This can be explained by the relatively high damping values (and lower resolution envelopes) used by Lalanne.

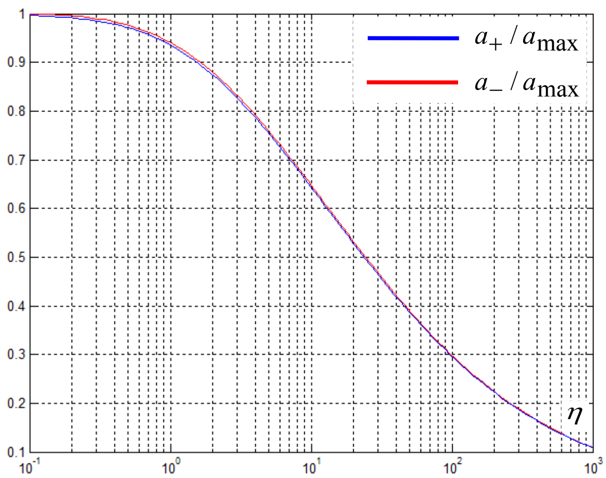


Fig. 5: Fraction of Maximum Amplitude

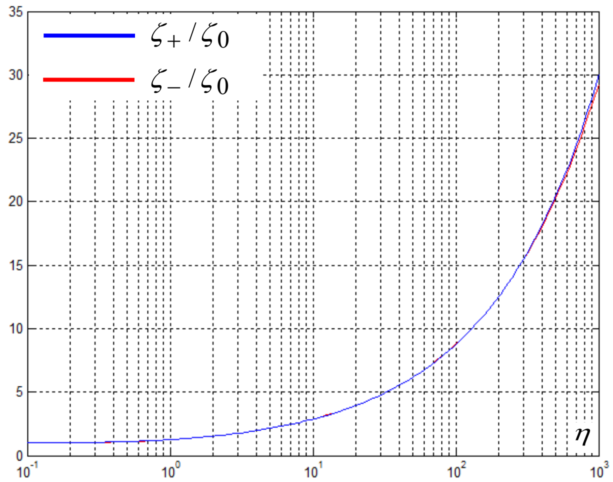


Fig. 6: Damping Ratio

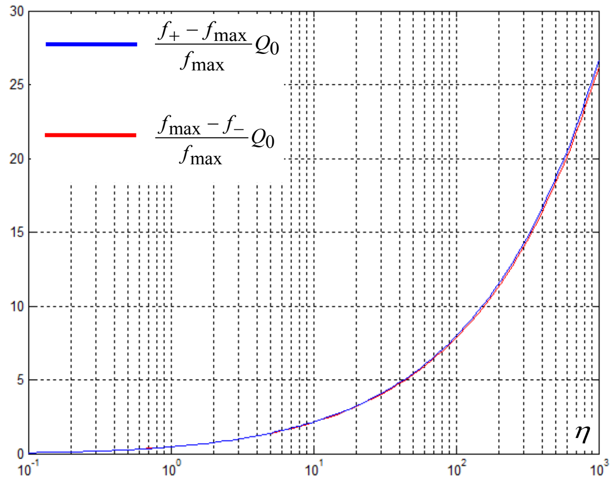


Fig. 7: Normalized Frequency Error

3.3 Relative Error Curves

The above master curves plotted as a function of η are not easy to interpret. To better understand and quantify the influence of the sweep rate on the different parameters, families of curves were generated from the master curves and plotted in Figures 8-11.

Each graph displays the relative error of the associated parameter for four different values of damping ($Q_0 = 10$, 25, 50, 100) as a function of the frequency f_0 (Hz) divided by the sweep rate R (oct/min). For a sweep rate of $R = 1$ oct/min, the x-axis displays frequencies. For other sweep rates the values of the x-axis are simply multiplied by R to obtain the corresponding frequency values.

The results for positive and negative sweep rates are shown in solid and dashed lines respectively. As noted previously, the differences between the two are small especially for low damping values.

The relative error for the effective transmissibility shown in Figure 11 is obtained by using $\tilde{T}_{\pm} = 2\zeta_{\pm}\, a_{\pm}$ along with the reference value of $\tilde{T}_{ij,0} = 1$.

For a given mode (f_0, ζ_0) and sweep rate R, the damping factor is by far the most sensitive parameter followed by the amplitude and effective transmissibility. The natural frequency is the least sensitive parameter.

As an example, using a sweep rate of 1 oct/min, a mode at 10 Hz with damping $\zeta_0 = 0.01$ will display a 2% error in frequency and over 70% error in damping.

The information provided by the relative error curves can be used in test preparation for the choice of sweep rates and the prediction of sweep rate effects on the modal parameters. The information can also be used in post-processing to adjust the modal parameters following modal identification. Software implementation is simple since all information can be derived from the data provided in Table 1.

Certain precautions should be taken when using this approach since the idealized behavior of the 1-DOF system may not correspond to the physical reality. Phenomena not taken into account include:

- Coupled modes (close peaks)
- Non-viscous sources of damping
- Nonlinear behavior
- Influence of noise

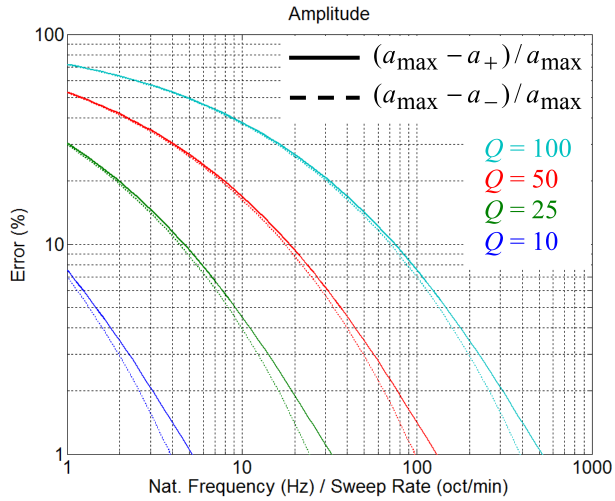


Fig. 8: Amplitude Errors

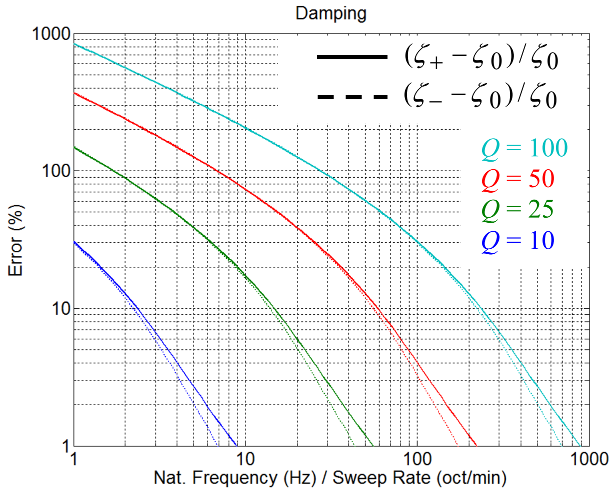


Fig. 9: Damping Errors

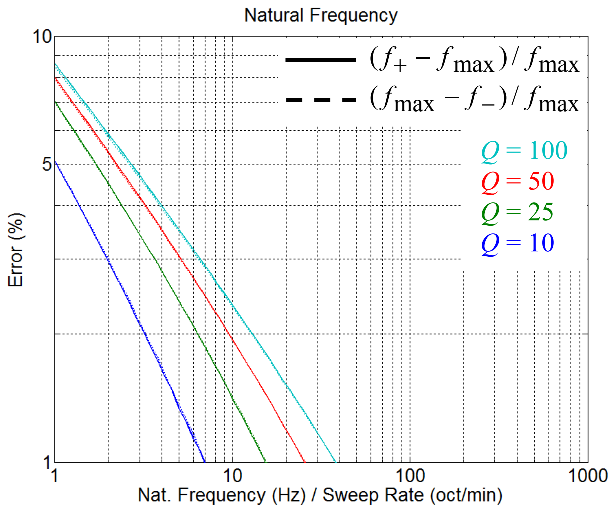


Fig. 10: Frequency Errors

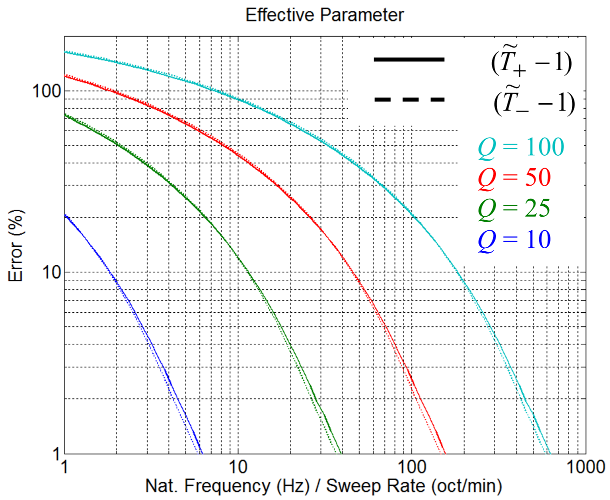


Fig. 11: Effective Parameter Errors

4. CONCLUSIONS

A consolidated approach for estimating the effect of sine sweep rates on modal parameters has been presented. The method is compatible with base-excitation vibration tests using both positive and negative exponential sine sweep rates.

The modal parameters include amplitude, natural frequency, damping and effective parameters. All information is derived from a small number of master curves expressed as a function of a non-dimensional sweep parameter.

The method can be used during test preparation for the selection of sweep rates and the prediction of sweep rate effects on critical modes. It can also be used in the context of modal identification to adjust the identified modes.

5. REFERENCES

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η	Q_0	a_+ / a_{max}	a_- / a_{max}	ζ_+ / ζ_0	ζ_- / ζ_0	$\frac{f_+ - f_{max}}{f_{max}} Q_0$	$\frac{f_{max} - f_-}{f_{max}} Q_0$
1.000e-01	9.304e+00	9.965e-01	9.997e-01	1.009e+00	1.006e+00	3.462e-02	8.857e-02
1.166e-01	1.005e+01	9.958e-01	9.991e-01	1.011e+00	1.007e+00	4.627e-02	9.634e-02
1.359e-01	1.085e+01	9.948e-01	9.984e-01	1.013e+00	1.009e+00	5.874e-02	1.063e-01
1.585e-01	1.171e+01	9.937e-01	9.976e-01	1.016e+00	1.011e+00	7.295e-02	1.177e-01
1.848e-01	1.265e+01	9.922e-01	9.964e-01	1.020e+00	1.014e+00	8.967e-02	1.303e-01
2.154e-01	1.366e+01	9.904e-01	9.948e-01	1.025e+00	1.019e+00	1.076e-01	1.455e-01
2.512e-01	1.475e+01	9.883e-01	9.928e-01	1.033e+00	1.025e+00	1.275e-01	1.639e-01
2.929e-01	1.592e+01	9.856e-01	9.902e-01	1.042e+00	1.035e+00	1.510e-01	1.850e-01
3.415e-01	1.719e+01	9.823e-01	9.872e-01	1.056e+00	1.047e+00	1.754e-01	2.069e-01
3.981e-01	1.856e+01	9.783e-01	9.834e-01	1.072e+00	1.063e+00	2.037e-01	2.332e-01
4.642e-01	2.004e+01	9.736e-01	9.788e-01	1.092e+00	1.083e+00	2.343e-01	2.615e-01
5.412e-01	2.164e+01	9.679e-01	9.733e-01	1.117e+00	1.108e+00	2.686e-01	2.948e-01
6.310e-01	2.337e+01	9.614e-01	9.669e-01	1.145e+00	1.137e+00	3.064e-01	3.305e-01
7.356e-01	2.523e+01	9.538e-01	9.594e-01	1.179e+00	1.171e+00	3.485e-01	3.704e-01
8.577e-01	2.725e+01	9.451e-01	9.507e-01	1.217e+00	1.210e+00	3.948e-01	4.161e-01
1.000e+00	2.942e+01	9.352e-01	9.407e-01	1.261e+00	1.255e+00	4.460e-01	4.661e-01
1.166e+00	3.177e+01	9.239e-01	9.296e-01	1.310e+00	1.304e+00	5.032e-01	5.193e-01
1.359e+00	3.430e+01	9.115e-01	9.172e-01	1.364e+00	1.360e+00	5.633e-01	5.796e-01
1.585e+00	3.704e+01	8.976e-01	9.033e-01	1.426e+00	1.422e+00	6.324e-01	6.469e-01
1.848e+00	3.999e+01	8.824e-01	8.881e-01	1.494e+00	1.491e+00	7.063e-01	7.187e-01
2.154e+00	4.318e+01	8.660e-01	8.716e-01	1.569e+00	1.567e+00	7.879e-01	7.978e-01
2.512e+00	4.663e+01	8.481e-01	8.538e-01	1.652e+00	1.651e+00	8.769e-01	8.869e-01
2.929e+00	5.035e+01	8.292e-01	8.346e-01	1.743e+00	1.743e+00	9.739e-01	9.805e-01
3.415e+00	5.437e+01	8.090e-01	8.144e-01	1.842e+00	1.844e+00	1.082e+00	1.085e+00
3.981e+00	5.870e+01	7.878e-01	7.930e-01	1.953e+00	1.955e+00	1.198e+00	1.198e+00
4.642e+00	6.339e+01	7.655e-01	7.707e-01	2.073e+00	2.077e+00	1.323e+00	1.323e+00
5.412e+00	6.844e+01	7.424e-01	7.474e-01	2.204e+00	2.209e+00	1.460e+00	1.458e+00
6.310e+00	7.390e+01	7.185e-01	7.234e-01	2.348e+00	2.353e+00	1.610e+00	1.604e+00
7.356e+00	7.980e+01	6.941e-01	6.988e-01	2.504e+00	2.512e+00	1.772e+00	1.763e+00
8.577e+00	8.616e+01	6.691e-01	6.737e-01	2.676e+00	2.683e+00	1.948e+00	1.937e+00
1.000e+01	9.304e+01	6.438e-01	6.483e-01	2.862e+00	2.871e+00	2.142e+00	2.128e+00
1.166e+01	1.005e+02	6.183e-01	6.226e-01	3.066e+00	3.075e+00	2.348e+00	2.332e+00
1.359e+01	1.085e+02	5.926e-01	5.968e-01	3.286e+00	3.297e+00	2.574e+00	2.552e+00
1.585e+01	1.171e+02	5.671e-01	5.710e-01	3.528e+00	3.540e+00	2.821e+00	2.793e+00
1.848e+01	1.265e+02	5.417e-01	5.455e-01	3.790e+00	3.803e+00	3.088e+00	3.053e+00
2.154e+01	1.366e+02	5.164e-01	5.201e-01	4.076e+00	4.090e+00	3.376e+00	3.337e+00
2.512e+01	1.475e+02	4.917e-01	4.952e-01	4.387e+00	4.402e+00	3.690e+00	3.647e+00
2.929e+01	1.592e+02	4.674e-01	4.707e-01	4.726e+00	4.740e+00	4.026e+00	3.978e+00
3.415e+01	1.719e+02	4.435e-01	4.467e-01	5.093e+00	5.110e+00	4.392e+00	4.334e+00
3.981e+01	1.856e+02	4.203e-01	4.233e-01	5.495e+00	5.511e+00	4.794e+00	4.726e+00
4.642e+01	2.004e+02	3.978e-01	4.007e-01	5.929e+00	5.946e+00	5.224e+00	5.148e+00
5.412e+01	2.164e+02	3.760e-01	3.787e-01	6.402e+00	6.418e+00	5.681e+00	5.603e+00
6.310e+01	2.337e+02	3.549e-01	3.575e-01	6.918e+00	6.934e+00	6.185e+00	6.098e+00
7.356e+01	2.523e+02	3.346e-01	3.370e-01	7.477e+00	7.491e+00	6.732e+00	6.629e+00
8.577e+01	2.725e+02	3.151e-01	3.174e-01	8.087e+00	8.099e+00	7.319e+00	7.208e+00
1.000e+02	2.942e+02	2.964e-01	2.986e-01	8.748e+00	8.759e+00	7.959e+00	7.830e+00
1.166e+02	3.177e+02	2.786e-01	2.807e-01	9.470e+00	9.471e+00	8.639e+00	8.503e+00
1.359e+02	3.430e+02	2.616e-01	2.635e-01	1.025e+01	1.025e+01	9.386e+00	9.229e+00
1.585e+02	3.704e+02	2.454e-01	2.472e-01	1.110e+01	1.109e+01	1.019e+01	1.002e+01
1.848e+02	3.999e+02	2.300e-01	2.317e-01	1.203e+01	1.202e+01	1.106e+01	1.087e+01
2.154e+02	4.318e+02	2.154e-01	2.170e-01	1.304e+01	1.301e+01	1.199e+01	1.179e+01
2.512e+02	4.663e+02	2.015e-01	2.030e-01	1.413e+01	1.410e+01	1.300e+01	1.279e+01
2.929e+02	5.035e+02	1.885e-01	1.899e-01	1.533e+01	1.527e+01	1.410e+01	1.385e+01
3.415e+02	5.437e+02	1.761e-01	1.775e-01	1.662e+01	1.655e+01	1.528e+01	1.502e+01
3.981e+02	5.870e+02	1.645e-01	1.657e-01	1.804e+01	1.792e+01	1.656e+01	1.627e+01
4.642e+02	6.339e+02	1.535e-01	1.547e-01	1.961e+01	1.944e+01	1.794e+01	1.762e+01
5.412e+02	6.844e+02	1.432e-01	1.443e-01	2.130e+01	2.110e+01	1.942e+01	1.908e+01
6.310e+02	7.390e+02	1.336e-01	1.346e-01	2.314e+01	2.286e+01	2.104e+01	2.067e+01
7.356e+02	7.980e+02	1.245e-01	1.254e-01	2.518e+01	2.482e+01	2.278e+01	2.237e+01
8.577e+02	8.616e+02	1.160e-01	1.168e-01	2.741e+01	2.694e+01	2.465e+01	2.421e+01
1.000e+03	9.304e+02	1.080e-01	1.088e-01	3.007e+01	2.924e+01	2.669e+01	2.619e+01

Table 1: Exponential Sine Sweep Response Characteristics