FAST SINE SWEEP AS AN ALTERNATIVE TO CLASSICAL SINE SWEEP FOR S/C QUALIFICATION

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ABSTRACT

The current practice to qualify spacecraft structures in the low frequency environment is the sine sweep test which requires the use of notching to avoid overtesting near the principal resonances. This practice is appropriate to cover sustained vibrations but not for short transients such as thrust transients which generate the most severe levels for the spacecraft primary structure.

This paper deals with the idea to replace the usual high level sine sweep test on shaker at system level by a low level one completed by a transient test in the same configuration, in order to be more representative of the real environment in amplitudes and durations, thus limiting unwanted overtesting. After presenting the context, the state of the art review reveals that the existing methods are not well adapted to the current needs whereas the proposed Fast Sine Sweep allows reproducing optimally shock spectra from Spacecraft / Launch Vehicle Coupled Loads Analysis. This new method is described in detail and followed by several examples to illustrate its efficiency.

1. CONTEXT

The classical sine sweep with notching is a practical and easy way to qualify spacecraft structures at low frequency. It also ensures for the launcher authority that the whole frequency band is covered without undertesting since the specified levels largely exceed the flight environment, except within some critical bands, and that any waiver (notching) shall be justified by the payload authority. This also requires a large amount of analysis for the project team to specify notchings and can induce a significant overtest in some frequency bands, as illustrated in Fig. 1. (from [1]) which compares applied test levels to Coupled Loads Analysis (CLA) results. This is mainly due to the modification of the environment where high level transient signals are replaced by a sustained sine sweep leading to a different distribution of responses associated with unrealistic durations. The real environment should always be reproduced as best as possible because equivalence criteria present intrinsic limitations.

Hence, overtesting could be highly attenuated by replacing the classical sine sweep by a transient test. The main problem here is the elaboration of the specification which is much more involved than the classical sine sweep. In order to overcome the difficulties inherent to the simple specification of transient signals, one can think about an adequate transformation of the time signal. This idea has already been presented in [2] and further developments have been made possible here thanks to a R&T study partially funded by CNES.

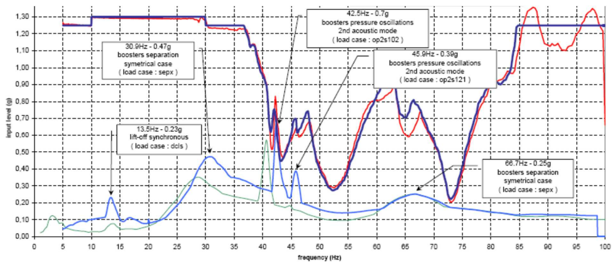


Figure 1. Example of longitudinal acceleration profiles Ariane 5 CLA (Q = 20)

Common practice makes use of the Shock Response Spectrum (SRS) which considers the effect of the transient signal on a Single degree of freedom (SDOF) system. The SRS represents the maximum response of a SDOF system according to its natural frequency for a given damping ratio (or *Q* factor). It has the advantages of representing transient severity and enabling envelopes.

Its main drawback is the loss of information as it includes only amplitude, leading to a lack of reversibility of this transformation. Thus, if only the amplitude is considered, a sine sweep lasting a few minutes can be representative of a one second transient, which is far from reality. The reason is that the amplitude alone cannot represent the whole severity of the transient; additional information is required (similar to the phase of the Fourier transform) to fully characterize the severity. The duration seems logical as a complementary information and previous studies [3] have developed the concept of duration spectrum which has proved tricky.

The context of CLA provides an interesting alternative through the definition of two SRS computed with two Q factors Q_1 and Q_2. The link with the duration comes from assuming that for a given frequency, the difference between the levels of the two SRS is proportional to the difference in duration. A larger difference in level means a higher duration at this frequency. These two spectra, containing nearly the same information as the Fourier spectrum, should lead to a certain degree of reversibility which will make the synthesized transient much more representative of the real environment. This reversibility might not be strictly ensured due to a certain redundancy of the two spectra which, on the other hand, should provide some margin to find a transient equivalent to the initial one in terms of its two SRS.

Under these conditions, this leads to a specification of the transient test through a SRS related to two Q factors derived from CLA results and possibly computed as a tailored envelope with adequate safety factors.

The selection of the Q factors should be representative of the specimen modal damping ratios. The usual values (Q = 20 and 40) may be a low range, while Q = 10 and 50 may be a maximum range.

The two SRS corresponding to two different Q factors must be consistent: their ratio must be between 1 and Q_2/Q_1. A ratio equal to Q_2/Q_1 will produce an infinite duration (classical sine sweep) while a lower ratio will reduce the durations in this proportion. This consistency is usually satisfied for two SRS related to a single transient but should be controlled in the case of envelopes.

Taking advantage of these considerations, the problem is now the shock synthesis method.

For a given SRS, several techniques already exist for shock synthesis, and details can be found in [4]. The general strategy of these methods relies on the combination of elementary waveforms such as damped or modulated sines. The next chapter details the state of the art in shock synthesis.

2. STATE OF THE ART

The question of shock synthesis has been widely studied, in particular for practical need to reproduce on a shaker a given shock defined by its SRS.

Current piloting systems include algorithms able to define a time signal representative of a specified SRS. For each frequency of the SRS, the algorithm generates an elementary waveform, typically a damped sine, whose SRS produces a peak whose amplitude depends on the damping of this waveform.

These elementary waveforms can then be combined along with a possible delay between them which enables, in a certain manner, to control the total shock duration. Obviously each elementary waveform has an influence on the surrounding frequencies of the SRS, and therefore the first synthesized SRS might be not sufficiently close to the reference. An iterative loop can then be applied to correct (in general) the amplitude of each elementary waveform to reach the reference SRS.

In this context of shock synthesis, some elementary waveforms have been found to be more or less efficient. Some of the most usual waveforms are presented hereafter. The results displayed in the figures are related to the following two cases:

- One flight event from Microscope S/C CLA results provided by CNES
- One SRS defined by three straight lines

The synthesized transient is displayed in the left figure and the corresponding SRS is displayed in red in the right figure superimposed with the reference SRS in blue. A value of Q = 20 is used to compute the SRS.

2.1. Compensated damped sine

The case of the simple damped sine has been first studied but presents some limitations: velocity and displacement are generally not zero at the end of the signal, which might make its replication on a shaker not possible. This problem can be solved by compensating the initial signal. Two efficient compensation techniques exist:

- Adding a highly damped sine to the global signal with a delay defined to compensate velocity and displacement
- Adding to each elementary waveform two exponential functions and a phase in the sine

The results of shock synthesis for both types of signal are presented in Fig. 2 and Fig. 3:

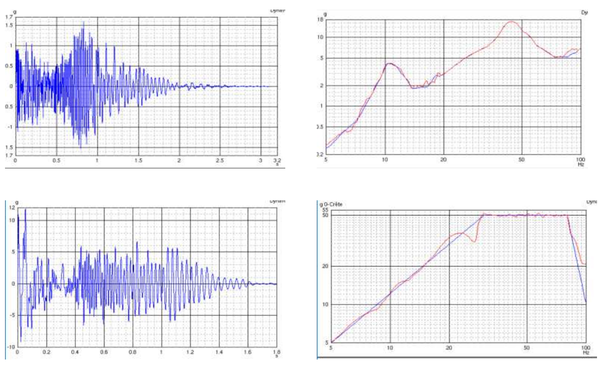


Figure 2. Shock synthesis with damped sine + sine

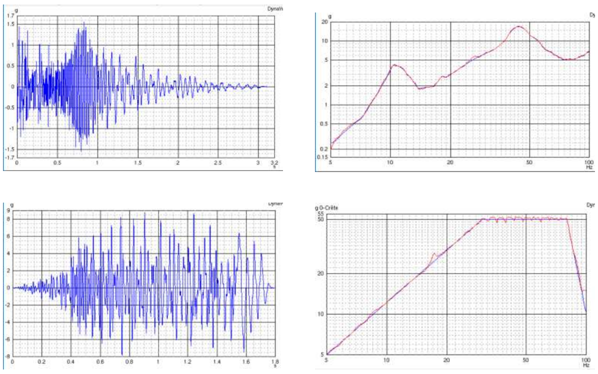


Figure 3. Shock synthesis with damped sine + 2 exponentials

2.2. ZERD function

An alternative to the compensated damped sine is the ZERD function (standing for ZEro Residual Displacement) defined by Eqs. 1-2:

$$a(t) = Ae^{-\eta\omega t}\left[\frac{1}{\omega}\sin(\omega t) - t\cos(\omega t + \phi)\right] \qquad (1)$$

$$\phi = \arctan\left(\frac{2\eta}{1-\eta^2}\right) \qquad (2)$$

The final transient is a combination of elementary ZERD functions at each circular frequency ω of the SRS.

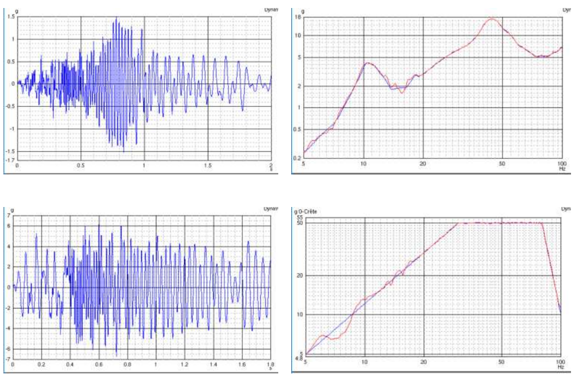


Figure 4. Shock synthesis with ZERD functions

2.3. WAVSIN function

The WAVSIN function can also be used as an elementary shape defined by Eq. 3:

$$a(t) = A\sin(2\pi bt)\sin(2\pi ft) \quad 0 \le t \le \tau \qquad (3)$$

Where $\tau = 1/2b$ and $f = Nb$, N being an odd integer greater than 1.
The first term is a half-sine of period 2τ and the second one defines N half-cycles of a sine of frequency f modulated by the first term. This elementary waveform produces zero residual velocity and displacement.
The parameter N_i allows modifying the shape and amplitude of the SRS peak of the elementary waveform at frequency f_i.

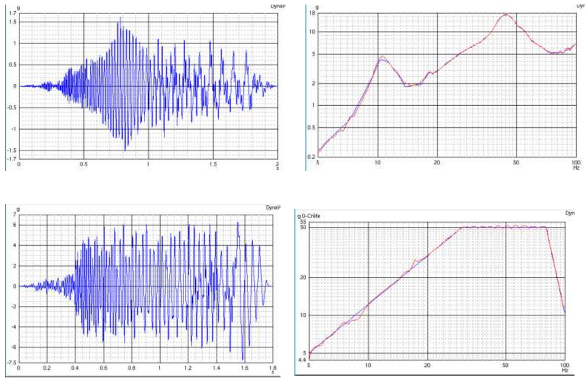


Figure 5. Shock synthesis with WAVSIN functions

Generally speaking, the elementary waveforms described in this paragraph seem to provide a good shock synthesis method for SRS including peaks. However, these shapes and this process do not seem well adapted to the case of a profiled SRS defined with straight lines. Moreover, the case of two SRS related to two Q factors is not covered by this shock synthesis method.
This is why a new synthesis method, still based on the equivalence in terms of SRS but adapted to the context of CLA in space industry, has been developed.

2.4. The Fast Sine Sweep

The concept of the Fast Sine Sweep (FSS) can be found in literature since it has already been studied. Some existing approaches are adapted to the simulation of a SRS computed with two values of the Q factor but address only the case of a SRS defined by a single slope or plateau in logarithmic scales and do not even consider the case of a profiled SRS comprising several straight lines.
This is why the new shock synthesis method based on the fast sine sweep developed in this paper presents several advantages in the present context.

3. NEW APPROACH FOR SHOCK SYNTHESIS

This new approach has been elaborated based on the following considerations:

- The main properties of the real environment should be preserved. This includes levels and durations which must be comparable to the real ones.
- The frequency content of the flight events should also be respected, which requires covering the entire frequency band considered. Techniques based on wavelets superimpose transients but in a discrete manner, which might prove of limited efficiency in the case of envelopes used for taking into account uncertainties and in standard specifications. Therefore a continuous approach should be considered by sweeping through frequencies, which yields a swept sine. However this sweep must be fast enough to respect durations to preserve the

transient property of the environment. This typically corresponds to a sweep rate of several octaves per *second.*

- Finally, the key point of the method is to consider this sweep rate as a function of time and thus of frequency. This provides a second parameter, which is complementary to the amplitude of the swept sine. Schematically, the amplitude will act on the average level of the two SRS and the sweep rate on their ratio.

The methodology to define a transient Fast Sine Sweep signal corresponding to a given SRS (or two SRS computed with two Q factors) is described hereafter.

4. METHODOLOGY

4.1. Introduction

A fast sine sweep can be expressed using the time function defined by Eq. 4:

$$\ddot{u}(t) = A(t) \sin E(t) \qquad (4)$$

With:

- $A(t)$ the sine modulation amplitude
- $E(t)$ the phase related to the sweep rate by Eq. 5:

$$\omega(t) = 2\,\pi\, f(t) = \frac{dE(t)}{dt} \quad V(t) = \frac{df(t)}{dt} \qquad (5)$$

$f(t)$ instantaneous frequency at instant t
$V(t)$ sweep rate (Hz/s) at instant t which can be defined in octaves per second by Eq. 6:

$$R(Oct/s) = \frac{R\,(oct/min)}{60} = \frac{V\,(Hz/s)}{f \ln 2} \qquad (6)$$

The two functions $A(t)$ and $V(t)$ (thus $E(t)$) are optimized in order to obtain the two SRS considered, denoted $S_1 = \text{SRS}(Q_1)$ and $S_2 = \text{SRS}(Q_2)$.
This optimization process, described in paragraph 4.2, requires an initial solution which can be obtained by considering an initial exponential sweep of constant rate. The amplitude attenuation at resonance is given in Fig. 6 as a function of parameter η defined by Eq. 7 and known as the dimensionless sweep rate:

$$\eta = Q^2\, V(Hz/s) = \frac{Q^2\, R(Oct/min)\, 60.\ln 2}{f} \qquad (7)$$

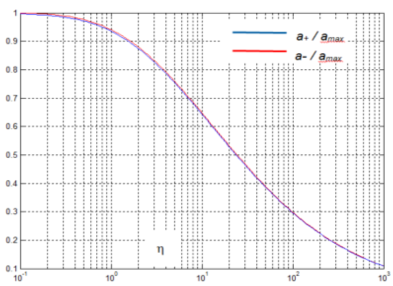


Figure 6. Attenuation curve – exponential sweep

Starting from this initial solution, the functions $A(t)$ and $V(t)$ (or $\eta(t)$) must be optimized to satisfy both SRS S_1 and S_2. This is the topic of the next paragraph.

4.2. Optimization strategy

The amplitude $A(f)$ and sweep rate $\eta(f)$ are expressed here as functions of frequency f discretized in N points. Thus, the total number of variables available for optimization is $2N$ and can be merged into a single vector x defined in Eq. 8:

$$x = \begin{bmatrix} A(f) \\ \eta(f) \end{bmatrix} \qquad (8)$$

For a given vector x, $S_1(x)$ and $S_2(x)$ are the respective SRS computed with $Q = Q_1$ and $Q = Q_2$.
The reference SRS associated with $S_1(x)$ and $S_2(x)$ are $\hat{S}_1$ and $\hat{S}_2$, assuming $Q_1 < Q_2$. The distance between the synthesis and the reference can be expressed by the error vector $f(x)$ in Eq. 9:

$$f(x) = \begin{bmatrix} S_1(x) - \hat{S}_1 \\ S_2(x) - \hat{S}_2 \end{bmatrix} \qquad (9)$$

Optimization using a single SRS can be performed by considering only the upper member of Eq. 9. The cost function F(x) is expressed in a quadratic form by Eq. 10:

$$F(x) = \frac{1}{2}\, f(x)^T f(x) \qquad (10)$$

To minimize $F(x)$, "non-linear least squares" methods may be used. These methods are based on the Taylor expansion of F(x) which is a function of the gradient vector g, and the Hessian and Jacobian matrices H and J in Eqs. 11-14:

$$F(x+h) = F(x) + h^T g + \frac{1}{2}\, h^T H h + O(\|h\|^3) \qquad (11)$$
$$g = F'(x) = J(x)^T f(x) \qquad (12)$$
$$H = F''(x) = J(x)^T J(x) \qquad (13)$$
$$J = \left[\frac{\partial f_i}{\partial x_j}(x)\right] \qquad (14)$$

The Levenberg-Marquardt optimization method is well adapted to this problem. It is based on Newton's method where the perturbation h is defined by cancelling the gradient g at $x+h$, but a parameter μ, named Marquardt parameter, is introduced in the problem, as expressed by Eq. 15:

$$(H + \mu I)h = -g \qquad (15)$$

This parameter allows adjusting the influence of the Hessian matrix H. A large value of μ is used when far from the minimum (risk of divergence) while a small value of μ is used when close to the minimum (similar to Newton's method). The value of parameter μ is automatically determined by the method.

Some constraints must be associated to this optimization problem:

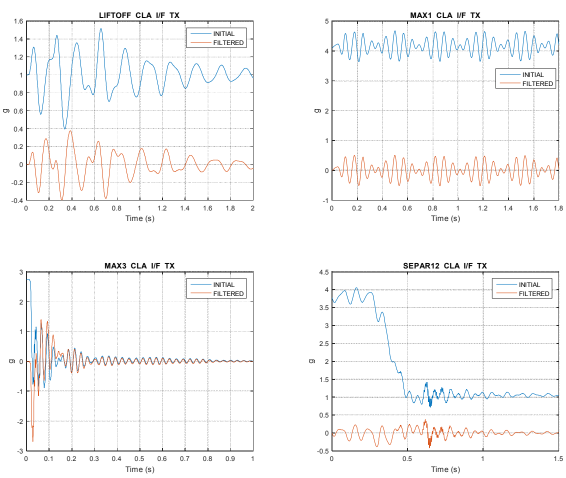
- The functions A and η must be positive
- The amplitude $A(f)$ must vary slowly with respect to the frequency; it should modulate only the swept sine envelope without disturbing the oscillations inside the envelope. Otherwise, spurious peaks might occur on the SRS. This constraint is introduced by defining the amplitude on a limited number of frequency points and by interpolating the other points by cubic smoothing.

This optimization method has been implemented in Matlab to generate a fast sine sweep matching one SRS or two SRS computed with two Q factors.

5. VALIDATION OF METHODOLOGY

5.1. Validation case

The methodology has been validated with the CLA results using the condensed model of the Microscope spacecraft. The interface is considered rigid, and the clamped model provides several modes up to 100 Hz.
The internal node IDEAS1 was used for the comparison of SRS and responses.
Viscous modal damping was used for the CLA.
The following flight events were considered for CLA: LIFTOFF, MAX1, MAX3, SEPAR12, SEPAR23.
The case of longitudinal X excitation was considered, highlighting the existence of quasi-static components coming mainly from thrust, engines ignition and cut-off, and often producing non zero initial conditions in the time signals.
For validation purposes, these components were filtered, as recommended in [1]. The raw (blue) and post-processed (red) interface longitudinal accelerations for each event are displayed in Fig. 7:



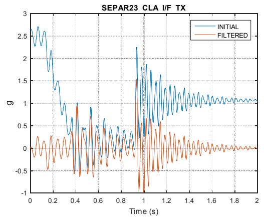


Figure 7. Interface TX accelerations

SRS were computed from the filtered interface accelerations for the 5 flight events with 2 damping values Q = 25 and Q = 50. The individual and enveloped SRS are displayed in Fig. 8:

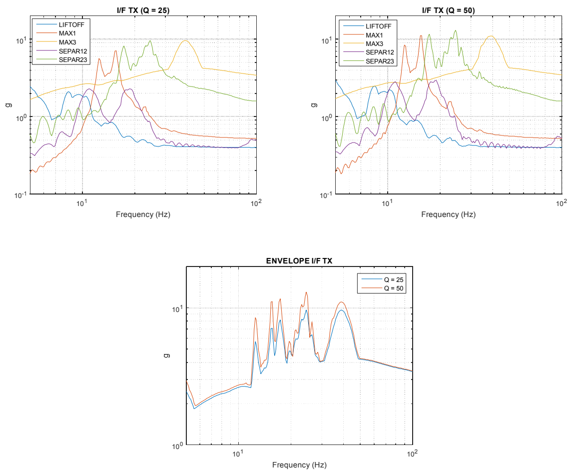


Figure 8. Interface TX SRS

For validation, two cases were considered:

- **Test case 1**: FSS from SRS of MAX3 event with Q = 25
- **Test case 2**: FSS from tailored SRS envelope with Q = 25 and Q = 50

5.2. Test case 1 – 1 event, 1 Q factor

The Fast Sine Sweep was synthesized from the SRS of the MAX3 event according to the methodology of chapter 4.
The FSS and filtered transient from CLA are displayed in Fig.9a. The durations of both signals are comparable (roughly 1 second). The corresponding SRS are displayed in Fig. 9b and are well correlated.
The responses at the internal node IDEAS1 were computed for both excitations (CLA and FSS) and are displayed in fig. 9c. They appear different but they present the same maximum value of 4.4g. The corresponding SRS plotted in Fig. 9d show that the FSS "covers" the CLA.
In order to compare the interface levels, the accelerations at Centre of Gravity (CoG) was computed for both excitations and are displayed in Fig. 9e, revealing similar maximum values. The SRS displayed in Fig. 9f show that the SRS from FSS covers the one from CLA, as in Fig. 9d, which is an important point from qualification point of view.

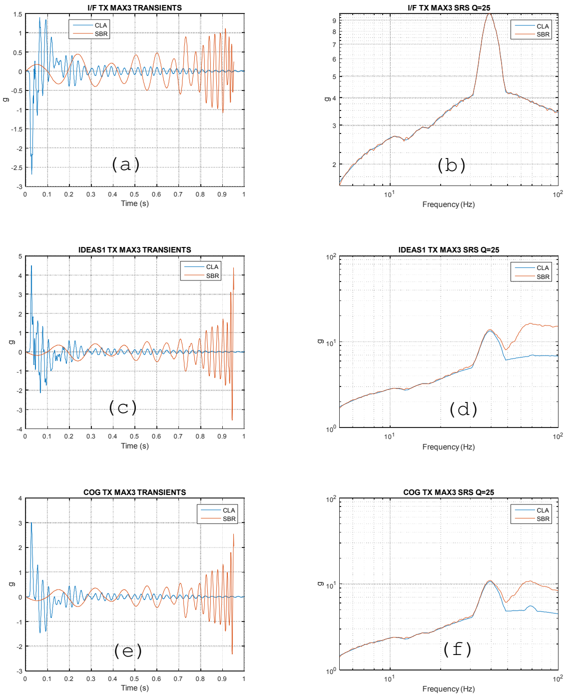


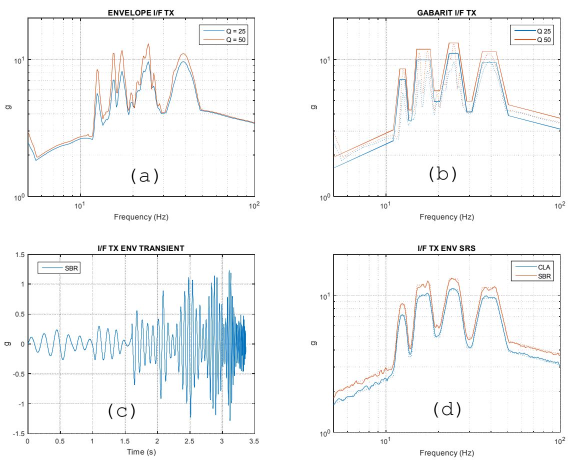
Figure 9. Test case 1 results

5.3. Test case 2 – envelope, 2 *Q* factors

This paragraph presents the case of the FSS synthesized from the envelope of the 5 events in the longitudinal direction and using 2 values of *Q*.

The envelopes of the SRS computed with *Q* = 25 and *Q* = 50 are plotted in Fig.10a. These spectra were tailored to produce two SRS defined by straight lines as plotted in Fig. 10b. The FSS was synthesized to match these two SRS and is displayed in Fig. 10c. The comparison of the SRS is shown in Fig. 10d, showing limited deviation on the whole frequency band.

For verification purpose, the SRS were also computed at internal node IDEAS1 and at the CoG. These results are displayed in Figs.10e to 10h which demonstrate that the SRS from FSS indeed cover the SRS for all events and both *Q* values, as in test case 1.



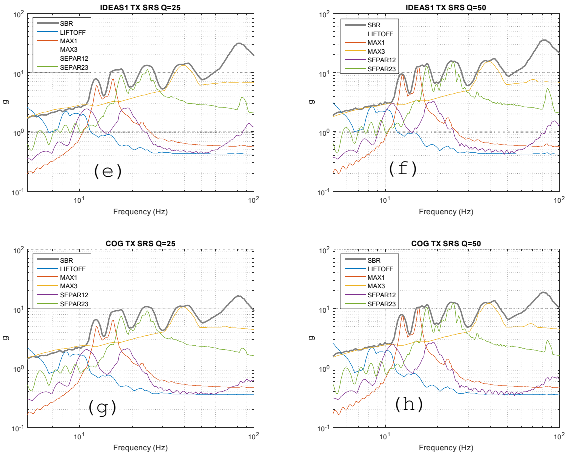


Figure 10. Test case 2 results

6. CONCLUSION

This paper presents a new methodology for shock synthesis that produces a transient signal in the form of a Fast Sine Sweep (FSS) capable of matching one or two SRS while maintaining similar amplitudes and durations. This new approach is particularly suited for CLA. The strength of the Fast Sine Sweep is its intrinsic continuous nature which can be used to match a single SRS with a given *Q* or 2 tailored SRS computed with 2 different *Q* factors. Future work will consist of experimental validation with a specimen on a shaker.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

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