A STOCHASTIC APPROACH TO THE VALIDATION OF SPACECRAFT STRUCTURAL DYNAMIC MODELS

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ABSTRACT

The validation of spacecraft dynamic models is often performed by a deterministic comparison of frequencies and mode shapes obtained from analysis and vibration testing. Unfortunately this approach cannot account for the scatter that is naturally present in the structure and whose influence on the structure's dynamic behavior can be considerable, and thus compromising the validation results.

A stochastic approach to dynamic model validation must therefore be considered. Although numerous stochastic techniques and tools exist today, it is necessary to adapt their use to the particular needs of spacecraft model validation. For example spacecraft testing is usually performed at a system level using a single specimen thus limiting the availability of experimentally measured uncertainty. Also, the need to use existing deterministic validation criteria such as the MAC must be taken into account to maintain compatibility with current practice.

This was the aim of the recently completed EDIS project [1] carried out under the technical management of the European Space Agency and performed by EADS CASA Espacio, EADS ASTRIUM, INTESPACE, MSC SOFTWARE and TOP MODAL. The project was carried out in three phases – methodology, software development and applications.

This paper presents an overview of the project along with a closer look at several of the technical findings including a novel stochastic modal identification tool. An assessment of the stochastic validation procedure is also presented along with suggestions for future studies.

1. INTRODUCTION

EDIS is the emanation, and in a certain sense, the consolidation of three previous projects: PROMENVIR, SCAT and RTMVI.

PROMENVIR [2] was a European ESPRIT project which succeeded in developing the first large-scale tool dedicated to *stochastic analysis* using Monte Carlo Simulation (MCS). Coupled to deterministic solvers, PROMENVIR generates meta-models or "response clouds" from which true system robustness may be assessed. Applications included automotive crash and satellite microvibrations.

SCAT [3] was a follow-up ESPRIT project whose main goal was to identify and implement methodologies for *stochastic model validation*. Stochastic metrics such as the Mahalanobis and Kolmogorov-Smirnov distances where implemented along with stochastic tools including PCA and correlation analysis. Perhaps the most important contribution of SCAT was the consolidation of a stochastic model improvement algorithm known as SDI (Stochastic Design Improvement) allowing the updating of a meta-model with respect to a target or reference meta-model.

RTMVI [4,5] was a ESA-funded project aimed at developing a set of methods and tools dedicated to the extraction of modal terms from the measured frequency response functions (FRFs) of base-driven vibration tests. Industrial constraints required an efficient and robust solution in order to allow for rapid (nearly real-time) identification of modes - often to be carried out between successive runs.

PROMENVIR, SCAT and RTMVI constitute the building blocks of the EDIS project whose goal was to implement a stochastic approach to the validation of dynamic models characterized by their vibrational modes.

One of the major objectives of the project was to develop and implement stochastic techniques for the validation of spacecraft structural dynamic models. An additional challenge came from the need to maintain compatibility with certain deterministic notions such as the MAC and mode pairing in order to provide a transitional and comparative link with current practice.

2. META-MODELS

A meta-model is a stochastic model represented by *p* input and output variables, *x* and *y*, whose values are sampled *n* times using either Monte Carlo Simulation (MCS) with an analytic model, or by physical measurement of the structure.

These values are stored in a *n* by *p* data matrix as depicted in Fig. 1.

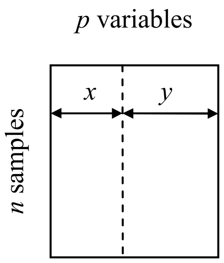


Fig. 1: Meta-model

In the context of the EDIS project, MCS was performed using the ST-ORM [6] software code (successor to PROMENVIR) coupled to finite element modal analysis using Nastran as illustrated in Fig. 2.

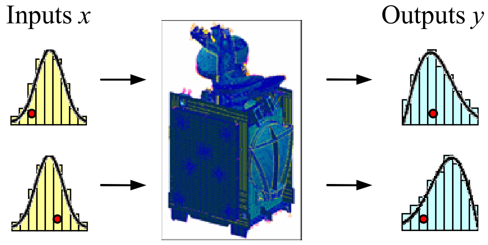


Fig. 2: Monte Carlo Simulation

The input variables, *x*, correspond to the design parameters of a spacecraft dynamic model such as plate thicknesses, beam section properties, material properties, local stiffnesses, etc.

As for the output variables, *y*, two possibilities were considered: a direct approach using FRFs, and a parametric approach using eigenmodes (natural frequencies, mode shapes, modal effective parameters, etc.). Following an evaluation, the modal approach was chosen over the FRF approach due to the existence of established comparison criteria such as the MAC (Modal Assurance Criterion) and natural frequency errors allowing a natural transition from deterministic to stochastic analysis.

Establishing a meta-model from test data normally involves the measurement of output variables using *n* different structures. This procedure may be feasible for standardized components which are produced in quantify such as reflectors or solar arrays. However system-level spacecraft structures such as telecom satellites are often produced as a single proto-flight specimen thus reducing the meta-model "cloud" to a single point.

To overcome this problem, a new stochastic modal identification method was developed and implemented allowing the extraction of a meta-model from a *single* physical specimen. This technique is presented after a discussion of model validation.

3. MODEL VALIDATION

One cannot validate a model without taking into account the effects of dispersion on the system's behavior. This point is illustrated below in Fig. 3 in which an experimental meta-model, *e*, and simulated meta-model, *s*, are depicted.

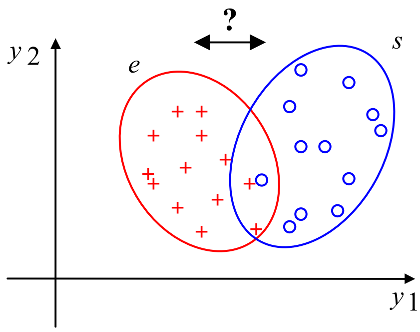


Fig. 3: Meta-model Validation

Each meta-model is plotted in the output space, *y*, as a collection of the measured and simulated (MCS) values known as scatter diagrams or anthill plots. In general, *y* may be of any dimension but is shown here in two dimensions for simplicity.

Deterministic model validation is analogous to considering one *single* point from each model and then computing the Euclidean distance, d_E, between them as defined in eq (1). Depending upon the chosen points, this distance could be larger or smaller and therefore may be unreliable.

$$d_E{}^2 = (y^e - y^s)^T (y^e - y^s) \qquad (1)$$

Stochastic model validation takes into account all the points of a meta-model leading to probabilistic distance metrics which naturally include the effects of dispersion.

The Mahalanobis distance, d_M, defined in eq (2) is one such metric which expresses the distance between the mean value of the variables (cloud centers) as a function of the dispersion (cloud size) obtained from the pooled covariance matrices, C. It is interesting to note the similarities between eqs (1) and (2) and in particular the role of the pooled covariance matrix, C, which transforms the physical units of d_E to the dimensionless units of d_M which may be interpreted as a *number of standard deviations* relative to C.

$$d_M{}^2 = (\bar{y}^e - \bar{y}^s)^T \, C^{-1} \, (\bar{y}^e - \bar{y}^s) \qquad (2)$$

4. RTMVI MODAL IDENTIFICATION

The dynamic behavior of a structure may be characterized by a set of frequency response functions (FRFs) each representing a transfer function between a specific excitation and response point.

If we consider the case of a single excitation point, e (e.g. a single vibrator for modal survey or a single axis base excitation vibration test) with measurements at various response points, r, then the general expression for the corresponding FRFs assuming mode superposition theory is given in eq (3)

$$X_{re}(\omega) = \sum_k A_k(\omega, \omega_k, \zeta_k) \tilde{X}_{re,k} \qquad (3)$$

comprising the following terms for each mode k

$A_k(\omega, \omega_k, \zeta_k)$	dynamic amplification
ω_k	circular natural frequency
ζ_k	modal damping factor
$\tilde{X}_{re,k}$	modal effective parameters

The above expression is valid for the three major types of transfer functions: accelerance (acceleration/force), transmissibility (acceleration/ acceleration) and dynamic mass (force/acceleration).

For all cases, the total response is expressed as a simple sum of the product of two terms: the dynamic amplification (a unitless complex valued term containing the natural frequency and damping factor), and the modal effective parameter [7,8] (a real valued purely modal term scaling the mode's contribution to the total response).

Therefore to completely characterize a given set of FRFs, one needs only identify three modal terms for each mode: the natural frequency, modal damping factor and the modal effective parameters.

All other modal terms such as the mode shape vector, generalized mass, and participation factors may be deduced from the modal effective parameters but are not essential to the modal identification process – nor to the model validation process as will be seen later.

Various methods of modal identification have been elaborated over the past several decades and may be divided into two general categories: time domain methods and frequency domain methods. Detailed information on these methods can be found in references such as Ewins [9] or Maia et al [10].

The RTMVI [4,5] method is a frequency domain method that was specifically developed to satisfy the needs related to spacecraft validation. Its formulation based on modal effective parameters of eq (3) is compatible with base excitation vibration tests as well as modal survey tests using applied forces.

It is a simple and efficient method that uses the *imaginary* part of the FRFs to provide a quick yet robust estimation of the underlying modes as illustrated in Fig. 4 and outlined below.

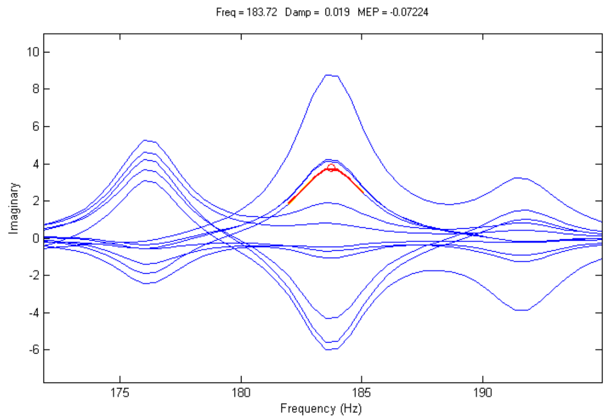


Fig. 4: RTMVI Modal Identification

Consider a segment of the imaginary part of a response surrounding a resonance or mode. From the amplitude and acuity of the segment, an estimation of the mode's natural frequency and damping can be obtained in an uncoupled (SDOF) manner.

For a given set of identified natural frequencies and damping factors, the remaining modal effective parameters may be obtained by solving the imaginary part of eq (3) as a coupled set of linear equations.

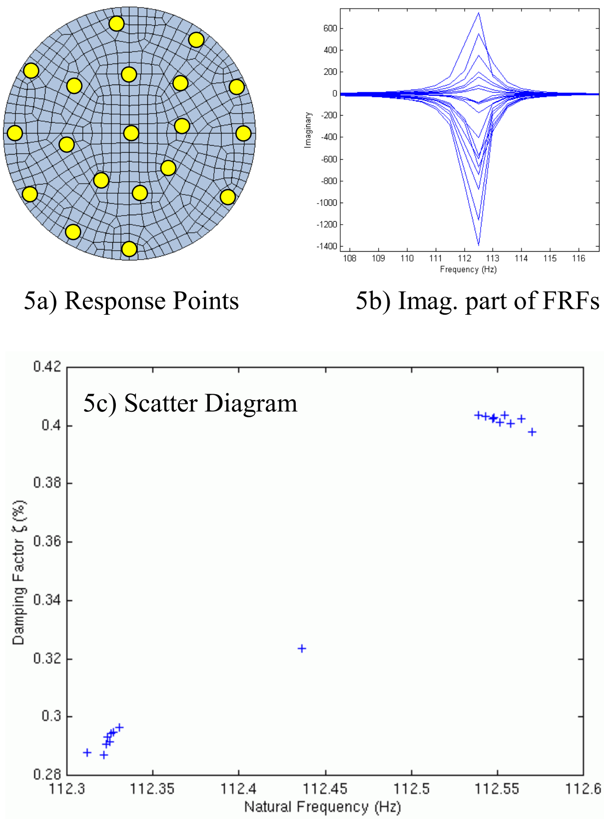
5. STOCHASTIC MODAL IDENTIFICATION

The RTMVI method described above is purely deterministic in that it furnishes a single natural frequency and damping value for each identified mode without any information on dispersion or uncertainties. However, a rather simple and direct transition to a stochastic meta-model can be made by establishing a relationship between response points and the samples of a meta-model.

Instead of using just one response to identify the natural frequency and damping of a given mode, *all* available responses are considered, one after the other, to obtain a *set* of natural frequencies and damping factors analogous to the samples (rows) of a meta-model.

The dispersion among these values indicates the presence of one or more underlying mechanisms such as nonlinearities, noise, influence of test set-up, etc.

As an illustration, consider the composite plate structure of Fig. 5 initially studied in the LARDAL project and used to validate the EDIS stochastic modal identification method. Accelerances were measured at 19 response points (Fig. 5a) of the plate suspended under a helium tent. The imaginary parts of the responses around the mode at 112 Hz are plotted in Fig. 5b from which a set of natural frequencies and damping factors are identified and plotted in the scatter diagram of Fig. 5c.



5a) Response Points　　5b) Imag. part of FRFs

Fig. 5: LARDAL Composite Plate

The dispersion pattern in the scatter diagram reveals a wealth of information. To begin with, we see that there are two very distinct clusters of values with nearly the same frequency ($\Delta f < 0.3$ Hz) yet substantially different damping factors ($\Delta\zeta > 0.1$). The clusters result from the fact that the FRFs were obtained from two different runs using 10 accelerometers per run and then changing their position between runs (with one accelerometer malfunctioning during one run).

The slight shift in frequency may be explained by the different effect of the added mass of the accelerometers on the give mode.

The difference in modal damping may be explained by a change in the helium content between runs. A change in helium content would also have an added mass effect which may or may not be negligible.

The LARDAL example demonstrates the importance of quantifying the dispersion associated with the identified modes in order to better understand the underlying mechanisms and provide a stochastic approach to model validation.

6. SOFTWARE INTEGRATION

The methods and tools described above have been integrated into a dedicated software package for dynamic model validation. The software combines four functional components: EDIS, ST-ORM, NASTRAN, and POST as shown in Fig. 6.

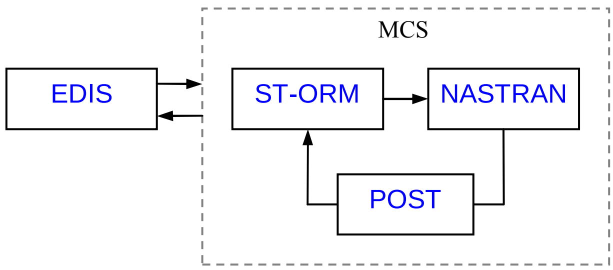


Fig. 6: EDIS Software Architecture

The EDIS software, entirely written in MATLAB, plays the role of a pre and post processor to the MCS block.

As a preprocessor, EDIS performs stochastic modal identification using the FRFs of a single vibration test as input. The output identified modes (experimental meta-model) are used during MCS to compute frequency errors and MAC values, and also by the EDIS postprocessor for model validation.

Monte Carlo Simulation is performed by ST-ORM which in turn calls upon NASTRAN to compute the normal modes (SOL 103) of the structure for each sampling of input variables. The choice of input variables and corresponding probability distributions is performed within the ST-ORM environment by editing the property entries of the bulk file.

The output variables are computed in the POST module (MATLAB) and written to a text "template file" which is read by ST-ORM in order to construct the analytic meta-model. The choice of output variables includes:

- Natural Frequencies
- Model Effective Parameters
- Errors in Natural Frequency
- Errors Modal Effective Parameter
- MAC

The last 3 outputs are computed using the experimental modes provided by the EDIS preprocessor. Mode matching is performed using the MAC as defined in eq (4) for two vectors u and v, or by using a modified version, MMAC, which penalizes vectors of different norms as shown in eq (5).

$$\text{MAC} = \frac{(u^{'}v)^2}{(u^{'}u)(v^{'}v)} \qquad (4)$$

$$\text{MMAC} = \frac{(u^{'}v)^2}{\max\left((u^{'}u)\,,(v^{'}v)\right)^2} \qquad (5)$$

The MMAC criterion, formulated in the EDIS project, is useful when comparing vectors of modal effective parameters (e.g. effective transmissibilities) or mode shapes with equal generalized mass. It is more severe than the MAC (MMAC$\leq$MAC) and unlike the MAC will effectively "filter" modes differing only by their magnitude.

Following MCS, the analytic meta-model is imported to the EDIS postprocessor which along with the experimental meta-model computes and displays the various distance metrics (deterministic and stochastic) for model validation purposes.

7. ACADEMIC EXAMPLE

The stochastic model validation software described above was first evaluated using a simple bending hinged beam model illustrated in Fig. 7.

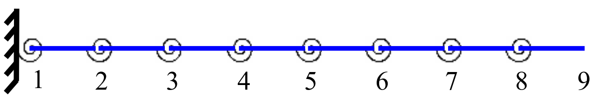


Fig. 7: Hinged Beam Model

The model is composed of 8 identical rigid beam elements with a total mass of 4000 kg connected by rotation springs with equal stiffness. The first four natural frequencies and effective masses are given in Fig. 8.

Mode	Nat. Freq. (Hz)	Eff. Mass (kg)
1	12.1	2593
2	76.0	779
3	214.3	254
4	421.9	119

Fig. 8: Analytic Hinged Beam Modes

Test data were constructed analytically by perturbing the beam model (spring 1 stiffness divided by 4, spring 4 stiffness multiplied by 3) and then computing a set of dynamic transmissibilities (one at each node) and the transverse dynamic mass. The frequencies and effective masses corresponding to the perturbed beam model are listed in Fig. 9.

Mode	Nat. Freq. (Hz)	Eff. Mass (kg)
1	10.4	2754
2	75.5	678
3	211.1	264
4	424.5	68

Fig. 9: "Test" Hinged Beam Modes

From the "test" FRFs, a "test" meta-model was derived using the stochastic modal identification tool of the EDIS preprocessor.

Next MCS using 50 runs was performed on the analytic model to generate an analytic meta-model. Normally distributed spring stiffnesses with a coefficient of variation, CV (standard deviation/mean) of 10% were introduced to characterize the dispersion of the input variables.

Finally, comparison of the "test" and analytic meta-models is performed in the EDIS postprocessor. The tabulated results showing Euclidean and Mahalanobis distances for natural frequencies and effective masses are presented in Fig. 10 along with a graphical representation from the EDIS postprocessor.

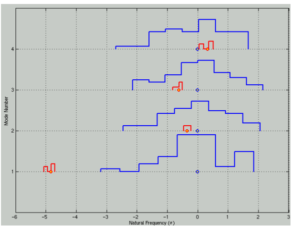
The frequencies and effective masses of each mode are also plotted as binned probability density functions in red and blue for the "test" and analytic models respectively.

Natural Frequencies

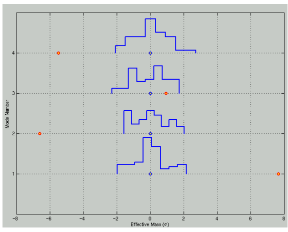
Mode No.	"Test" Mean	"Test" CV	Analytic Mean	Analytic CV	Euclid %	Mahal (std)
1	10.40	0.52	11.99	2.74	15.3	4.8
2	75.13	0.27	75.74	2.31	0.8	0.4
3	210.70	0.28	213.6	2.22	1.4	0.6
4	423.59	0.50	420.7	2.13	-0.7	0.3

Effective Masses

Mode No.	"Test" Mean	"Test" CV	Analytic Mean	Analytic CV	Euclid %	Mahal (std)
1	2754	-	2592	0.82	-6.3	7.7
2	681	-	779	1.94	12.6	6.5
3	265	-	255	4.61	-3.9	0.9
4	69	-	119	7.68	42.1	5.5



Natural Frequencies



Effective Masses

Fig. 10: Initial Meta-model Comparison

We can see from Fig. 10 a significant error in the natural frequency of the first mode (15 % Euclidean error and nearly 5 standard deviations for the Mahalanobis distance). Moreover, all of the effective masses with the exception of the third mode show significant error.

Using the SDI (Stochastic Design Improvement) algorithm in ST-ORM, the first four frequencies and effective masses of the analytic model were updated by allowing stiffness modifications to all of the rotational springs. The resulting modifications to the spring stiffnesses are presented in Fig. 11 and concord with the perturbed values used to generate the "test" model.

Spring	Stiffness		
	Analytic	"Test"	Updated
1	3.00e+8	1.50e+8	1.47e+8
2	3.00e+8	3.00e+8	3.13e+8
3	3.00e+8	3.00e+8	2.73e+8
4	3.00e+8	9.00e+8	7.38e+8
5	3.00e+8	3.00e+8	2.73e+8
6	3.00e+8	3.00e+8	2.71e+8
7	3.00e+8	3.00e+8	3.29e+8
8	3.00e+8	3.00e+8	3.00e+8

Fig. 11: Updated Spring Stiffnesses

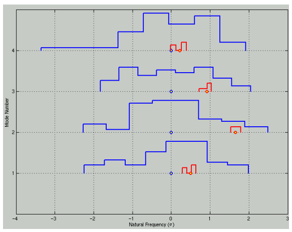
A final comparison was performed between the "test" meta-model and the updated analytic meta-model. The results are presented in Fig. 12.

Natural Frequencies

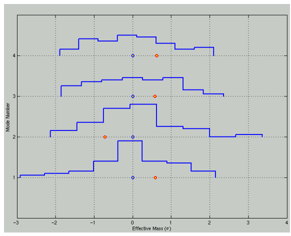
Mode No.	"Test" Mean	"Test" CV	Analytic Mean	Analytic CV	Euclid %	Mahal (std)
1	10.40	0.52	10.23	3.20	-1.6	0.5
2	75.13	0.27	72.39	2.29	-3.6	1.7
3	210.70	0.28	206.4	2.28	-2.0	0.9
4	423.59	0.50	421.3	2.49	-0.6	0.2

Effective Masses

Mode No.	"Test" Mean	"Test" CV	Analytic Mean	Analytic CV	Euclid %	Mahal (std)
1	2754	-	2743	0.67	-0.4	0.6
2	681	-	691	2.14	1.5	0.6
3	265	-	259	4.00	-2.3	0.6
4	69	-	66	7.43	4.5	0.6



Natural Frequencies



Effective Masses

Fig. 12: Final Meta-model Comparison

From Fig. 12 we see that the Mahalanobis distances are consistently less than 1 standard deviation with the exception of the second natural frequency. This is corroborated graphically by noticing that the experimental values in red are located well inside the distribution of the analytic values in blue.

Furthermore it is interesting to compare the Euclidean and Mahalanobis distances. For example, the distance between the natural frequencies of mode 4 is relatively small according to both metrics. Inversely, the effective masses for the same mode result in a relatively large Euclidean distance yet a relatively small Mahalanobis distance.

The advantage of the Mahalanobis distance is that it expresses the error between models as a function of the inherent scatter in the system thus providing a robust model validation criterion.

8. INDUSTRIAL APPLICATION

An industrial application involving a Eurostar E3000 telecom satellite was performed as a final validation of the methods and software.

The Nastran finite element model of the structure is illustrated in Fig. 13 and includes as principal appendages, two antenna reflectors, two solar arrays, a top floor antenna and two batteries. The total modeled mass (with empty tanks) is 1990 kg.

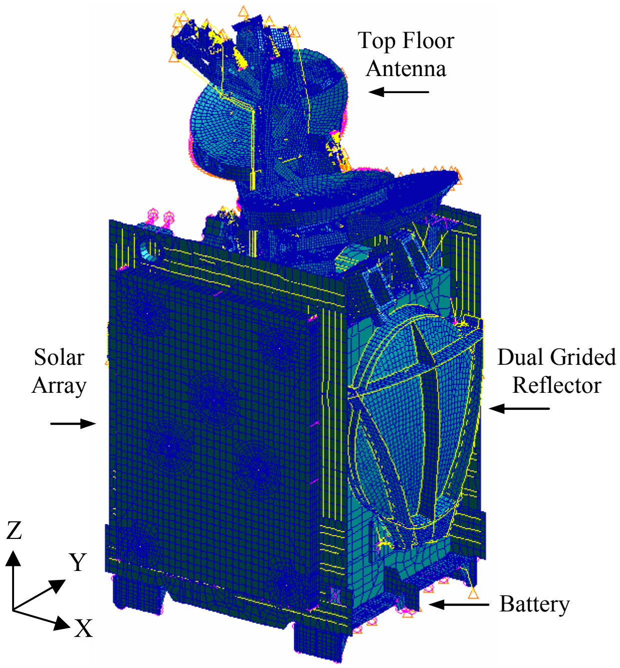


Fig. 13: Eurostar E3000 Spacecraft

A base excitation vibration test on a slip table along the lateral x-direction provided a set of dynamic transmissibilities measured at 264 accelerometer locations. No reaction force (effective mass) measurements were performed.

The imaginary parts of the experimental FRFs are displayed in Fig. 14 with a description of the first 8 prominent resonances.

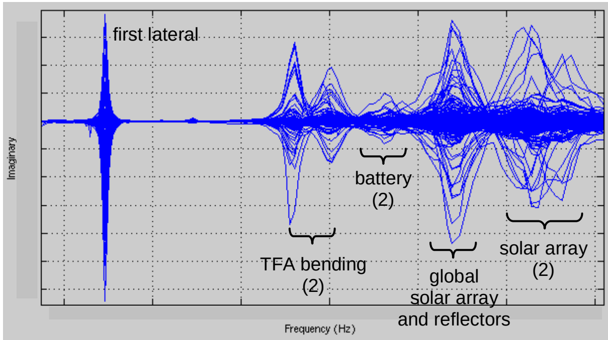


Fig. 14: Eurostar E3000 FRFs

The next step was to extract the 8 modes using the stochastic modal identification tool. The resulting dispersion in natural frequency and damping for each of the modes is displayed in Fig. 15.

Mode	CV (%)	
	Nat. Freq.	Damping
1	0.17	22.41
2	0.36	25.07
3	0.79	27.09
4	0.65	61.86
5	0.43	55.21
6	0.58	38.05
7	0.45	49.90
8	0.85	43.89

Fig. 15: Dispersion in Experimental Modes

The natural frequencies have a consistently small level of scatter (CV < 1%) despite relatively course frequency intervals in the FRFs. On the other hand, high levels of scatter were detected in the damping factors – especially beyond the third mode where values of CV are close to 50 %.

Although damping factors are not directly involved in the validation process, it is important to note that FRF response levels are inversely proportional to the damping and therefore it is logical to assume that high scatter in damping may be indicative of high scatter in the experimental mode shape components.

In order to perform an initial comparison, MCS Simulation was carried out on the finite element model using as stochastic input variables the interface stiffnesses of the different appendages. This choice was motivated by the fact that structural junctions are often represented by simple elements whose stiffness characteristics are difficult to estimate.

Natural frequency errors and MAC values are tabulated in Fig. 16. Mahalanobis distances are not shown due to stability problems with mode shape pairing and subsequent dispersion levels which were encountered during the MCS. These problems were later resolved by defining a group of sensors for each experimental mode and limiting the mode pairing to the sensor groups.

Mode	Freq Error (%)	MAC
1	3.66	0.93
2	-3.37	0.78
3	-6.16	0.93
4	32.35	0.18
5	10.47	0.80
6	-8.87	0.04
7	11.09	0.21
8	-9.59	0.19

Fig. 16: E3000 Initial Comparison

The frequency errors for the first three modes in Fig. 16 are relatively low with very good MAC values. Mode 5 is the only other well paired mode with a MAC of 0.8 and a frequency error of about 10 %.

Several detailed correlation studies were carried out in ST-ORM using Pearson and Spearman coefficients in order to identify a suitable set of input variables for model updating.

Next, three updating campaigns using the SDI algorithm were performed. In addition to the appendage interface stiffnesses, the central cone cylinder thicknesses and Top Floor Antenna plate thicknesses were added as input variables.

The results of the final comparison using the updated model are given below in Fig. 17.

Mode	Frequency Errors		MAC
	Euclid (%)	Mahal (std)	
1	0.17	0.25	0.93
2	-0.73	0.55	0.79
3	-3.94	3.91	0.84

Fig. 17: E3000 Final Comparison

Significant improvement is attained for the first two modes resulting in small frequency errors for both Euclidean and Mahalanobis distances and nearly unchanged MAC values. The third mode shows improvement in frequency but with a slight degradation in the MAC.

The inability to update the third mode is not the fault of the stochastic approach in itself, but rather related to certain characteristics of the model as summarized below.

- The models of the various appendages are not sufficiently accurate resulting in system-level behavior that is difficult to improve. A gradual process composed of separate subsystem updates followed by a system-level update would probably produce better results.
- The definition of input variables including their range of variation is mainly guided by the engineer's experience and knowledge of the model. Considering the complexity of a system-level model, this manual approach quickly becomes burdensome and most likely inaccurate.
- Equivalent or simplified modeling techniques are often employed, particularly at interfaces. These modeling techniques should be introduced as additional inputs to the stochastic validation process rather than being rigidly imposed from the beginning.

9. CONCLUSIONS

Stochastic model validation takes into account the uncertainties associated with the input variables of the finite element model as well as the scatter in test data. Moreover, through MCS input/output relationships may be examined over the entire design space instead of being limited to a localized gradient.

A stochastic model validation software code integrating existing tools as well as new tools has been developed and successfully evaluated. The code uses accepted modal criteria such as frequencies, model effective parameters and MAC values while introducing stochastic metrics based on the Mahalanobis distance. The code also includes a novel stochastic modal identification tool.

Several enhancements to model validation (both stochastic and deterministic) were identified and proven useful. They include the use of a modified MAC which penalizes vectors of different magnitude, and the use of sensor groups to facilitate and improve mode matching.

The following topics for future development were identified during the course of the project.

- Determining the effects of the high scatter in damping on mode shapes and in particular on the MAC values.
- Combining modes in areas of high modal density to improve comparison of response levels and mode pairing.
- Implementing a substructure approach to model validation in order to reduce complexity and facilitate the updating of the interface properties.

10. ACKNOWLEDGEMENTS

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11. REFERENCES

1. Calvi, A., Garcia de Parades, S., Roy, N. and Lefevre, Y-M., On the Development of a Stochastic Approach for the Validation of Spacecraft Structural Dynamic Models, European Conference on Spacecraft Structures, Materials and Mechanical Testing, Toulouse, Dec. 11-13, 2002.

2. Marczyk, J. (editor), Computational Stochastic Mechanics in a Meta-Computing Perspective, *CIMNE Publication*, Barcelona, Spain, 1997.

3. Doltsinis, I. (editor), Stochastic Analysis of Multivariate Systems in Computational Mechanics and Engineering, *CIMNE Publication*, Barcelona, Spain, 1999.

4. Real time modal vibration identification for communication satellite testing, ESTEC Invitation to Tender AO3458, March 1999.

5. Girard A., Roy N., Guyot M., Bugeat L.-P., Modal Identification via Effective Parameters: An Industrial Solution, European Conference on Spacecraft Structures, Materials and Mechanical Testing, Proceedings ESA SP-468, p. 283-288, March 2001.

6. A Meta-Computing Simulation Environment for Stochastic Optimization and Robustness Management, ST-ORM User's Manual Version 2.2, EASi Engineering GmbH, Germany, 2002.

7. Girard A. and Roy N. A., Modal Effective Parameters in Structural Dynamics, *Revue Européenne des Eléments Finis*, Vol. 6, n° 2, p. 233-254, 1997

8. Girard, A. and Roy, N., Dynamique des structures industrielles, *Lavoisier*, 2003.

9. Ewins, D., Modal Testing: Theory and Practice, *Research Studies Press*, Ed. 2, 2000.

10. Maia, N.N.M et al, Theoretical and Experimental Modal Analysis, *Research Studies Press LTD*, 1997.