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IMPACT OF RESIDUAL MODES IN STRUCTURAL DYNAMICS

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- Frequency Response Functions
- Residual Modes
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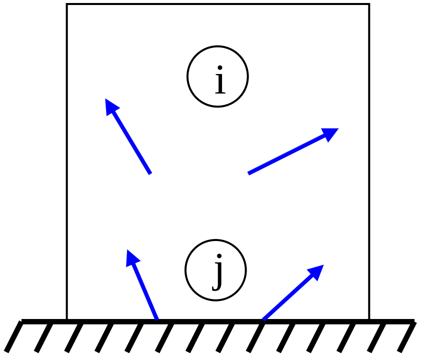
- Conclusions

FRFs / Equations of Motion



$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ij} \\ \mathbf{M}_{ji} & \mathbf{M}_{jj} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_j \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{ii} & \mathbf{C}_{ij} \\ \mathbf{C}_{ji} & \mathbf{C}_{jj} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}_i \\ \dot{\mathbf{u}}_j \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ij} \\ \mathbf{K}_{ji} & \mathbf{K}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \end{bmatrix} = \begin{bmatrix} \mathbf{F}_i \\ \mathbf{F}_j \end{bmatrix}$$

(i) internal DOFs ⇨ Excitation : $\mathbf{F}_i$
Response : $\mathbf{u}_i , \dot{\mathbf{u}}_i , \ddot{\mathbf{u}}_i$

(j) junction DOFs ⇨ Excitation : $\mathbf{u}_j , \dot{\mathbf{u}}_j , \ddot{\mathbf{u}}_j$
Response : $\mathbf{F}_j$

MODAL PROJECTION

internal motion	=	junction induced motion	+	relative motion
$\mathbf{u}_i$	$=$	$\mathbf{\Psi}_{ij} \quad \mathbf{u}_j$	$+$	$\mathbf{\Phi}_{ik} \quad \mathbf{q}_k$

JUNCTION MODES (↑ $\mathbf{\Psi}_{ij}$) NORMAL MODES (↑ $\mathbf{\Phi}_{ik}$)

FRFs / Junction and Normal Modes

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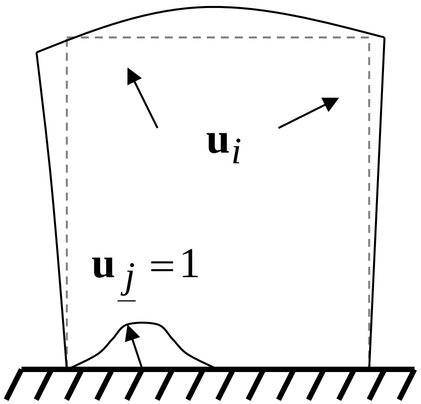
JUNCTION MODES

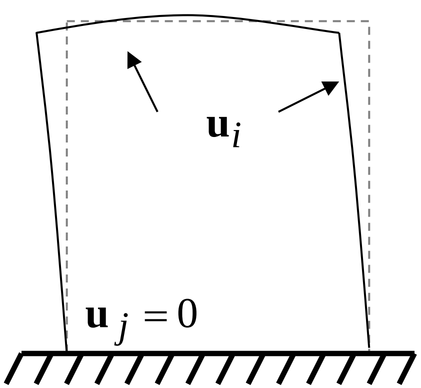


$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ij} \\ \mathbf{K}_{ji} & \mathbf{K}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{\Psi}_{ij} \\ \mathbf{I}_{jj} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{ij} \\ \mathbf{F}_{jj} \end{bmatrix} \Rightarrow \mathbf{\Psi}_{ij} = -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ij}$$

imposed unit displacements

- static deformations
- rigid junction → rigid body modes

NORMAL MODES



$$\left(-\omega_{\underline{k}}^2 \mathbf{M}_{ii} + \mathbf{K}_{ii}\right)\mathbf{\Phi}_{i\underline{k}} = \mathbf{0}$$ (conservative system)

$\omega_{\underline{k}}^2$	N eigenvalues	$f_{\underline{k}} = \omega_{\underline{k}} / 2\pi$
$\mathbf{\Phi}_{i\underline{k}}$	N eigenvectors	

FRFs / Transformed Equations of Motion

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$$\mathbf{u}_{i+j} = \begin{bmatrix} \mathbf{\Phi}_{(i+j)k} & \mathbf{\Psi}_{(i+j)j} \end{bmatrix} \begin{bmatrix} \mathbf{q}_k \\ \mathbf{u}_j \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{ik} & \mathbf{\Psi}_{ij} \\ \mathbf{0}_{jk} & \mathbf{I}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{q}_k \\ \mathbf{u}_j \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{m}_{kk} & \mathbf{L}_{kj} \\ \mathbf{L}_{jk} & \overline{\mathbf{M}}_{jj} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_k \\ \ddot{\mathbf{u}}_j \end{bmatrix} + \begin{bmatrix} \mathbf{c}_{kk} & \mathbf{0}_{kj} \\ \mathbf{0}_{jk} & \mathbf{0}_{jj} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_k \\ \dot{\mathbf{u}}_j \end{bmatrix} + \begin{bmatrix} \mathbf{k}_{kk} & \mathbf{0}_{kj} \\ \mathbf{0}_{jk} & \overline{\mathbf{K}}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{q}_k \\ \mathbf{u}_j \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{ki}\,\mathbf{F}_i \\ \mathbf{\Psi}_{ji}\,\mathbf{F}_i + \mathbf{F}_j \end{bmatrix}$$

$\mathbf{m}_{kk}$	diagonal matrix of generalized masses
$\mathbf{c}_{kk}$	matrix of generalized damping, *a priori* coupled → modal damping $\zeta_{\underline{k}} = \dfrac{c_{\underline{k}}}{2\sqrt{k_{\underline{k}}\, m_{\underline{k}}}}$
$\mathbf{k}_{kk}$	diagonal matrix of generalized stiffnesses
$\mathbf{L}_{kj}$	matrix of participation factors (coupling $\mathbf{\Phi}_{ik} \Leftrightarrow \mathbf{\Psi}_{ij}$)
$\overline{\mathbf{M}}_{jj}$	condensed mass matrix (rigid body mass matrix for rigid junction)
$\overline{\mathbf{K}}_{jj}$	condensed stiffness matrix (null for rigid junction)

FRFs / Frequency Response Functions

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$$\begin{bmatrix} \mathbf{u}_i(\omega) \\ \mathbf{F}_j(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{ii}(\omega) & \mathbf{T}_{ij}(\omega) \\ -\mathbf{T}_{ji}(\omega) & -\omega^2\,\mathbf{M}_{jj}(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{F}_i(\omega) \\ \mathbf{u}_j(\omega) \end{bmatrix}$$

dynamic amplification

dynamic flexibility

$$\mathbf{G}_{ii}(\omega) = \sum_{\underline{k}=1}^{N} H_{\underline{k}}(\omega)\, \tilde{\mathbf{G}}_{ii,\underline{k}}$$

dynamic transmissibility

$$\mathbf{T}_{ij}(\omega) \approx \sum_{\underline{k}=1}^{N} T_{\underline{k}}(\omega)\, \tilde{\mathbf{T}}_{ij,\underline{k}}$$

dynamic mass

$$\mathbf{M}_{jj}(\omega) \approx \sum_{\underline{k}=1}^{N} T_{\underline{k}}(\omega)\, \tilde{\mathbf{M}}_{jj,\underline{k}}$$

$$\Rightarrow \quad \mathbf{X}(\omega) = \sum_{\underline{k}=1}^{N} A_{\underline{k}}(\omega)\, \tilde{\mathbf{X}}_{\underline{k}}$$

modal effective parameters

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FRFs / Dynamic Amplification

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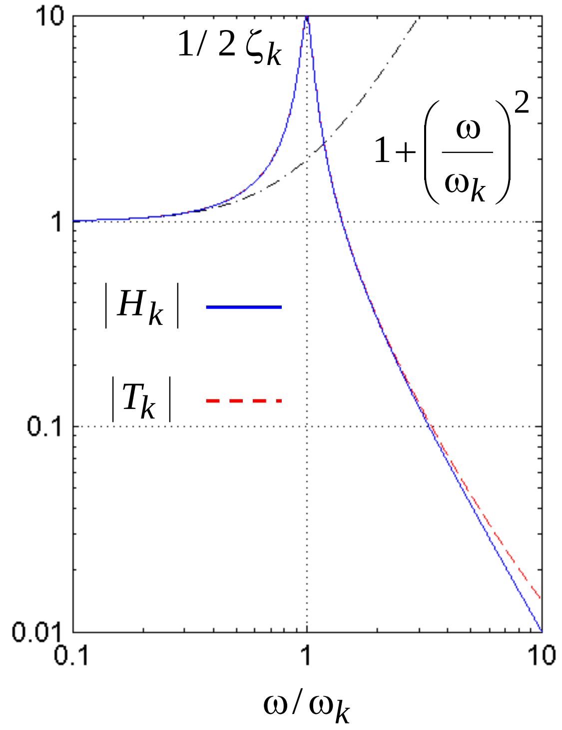
$$H_k(\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_k}\right)^2 + i\,2\,\zeta_k \left(\frac{\omega}{\omega_k}\right)}$$

$$T_k(\omega) = \frac{1 + i\,2\,\zeta_k \left(\frac{\omega}{\omega_k}\right)}{1 - \left(\frac{\omega}{\omega_k}\right)^2 + i\,2\,\zeta_k \left(\frac{\omega}{\omega_k}\right)}$$



$$A_k(\omega) = H_k(\omega)\,,\ T_k(\omega) \approx 1 + \left(\frac{\omega}{\omega_k}\right)^2$$ Taylor series expansion

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FRFs / Modal Effective Parameters

Summation Rules

	static	inertia
$\tilde{\mathbf{X}}$	$\overline{\mathbf{X}} = \sum_{\underline{k}=1}^{N} \tilde{\mathbf{X}}_{\underline{k}}$	$\overline{\mathbf{Y}} = \sum_{\underline{k}=1}^{N} \tilde{\mathbf{X}}_{\underline{k}} / \omega_{\underline{k}}^{2}$
$\tilde{\mathbf{G}}_{ii,\underline{k}}$	$\mathbf{G}_{ii} = \mathbf{K}_{ii}^{-1}$	$\mathbf{G}_{ii}\, \mathbf{M}_{ii}\, \mathbf{G}_{ii}$
$\tilde{\mathbf{T}}_{ij,\underline{k}}$	$\hat{\mathbf{\Psi}}_{ij} = \mathbf{\Psi}_{ij} + \mathbf{M}_{ii}^{-1}\, \mathbf{M}_{ij}$	$\mathbf{G}_{ii}\, \mathbf{M}_{ii}\, \hat{\mathbf{\Psi}}_{ij}$
$\tilde{\mathbf{M}}_{jj,\underline{k}}$	$\hat{\mathbf{M}}_{jj} = \hat{\mathbf{\Psi}}_{ji}\, \mathbf{M}_{ii}\, \hat{\mathbf{\Psi}}_{ij}$	$\hat{\mathbf{\Psi}}_{ji}\, \mathbf{M}_{ii}\, \mathbf{G}_{ii}\, \mathbf{M}_{ii}\, \hat{\mathbf{\Psi}}_{ij}$
residual terms ($n \leq N$)	$\mathbf{X}_{res} = \overline{\mathbf{X}} - \sum_{\underline{k}=1}^{n} \tilde{\mathbf{X}}_{\underline{k}}$	$\mathbf{Y}_{res} = \overline{\mathbf{Y}} - \sum_{\underline{k}=1}^{n} \tilde{\mathbf{X}}_{\underline{k}} / \omega_{\underline{k}}^{2}$

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FRFs / Mode Truncation Corrections

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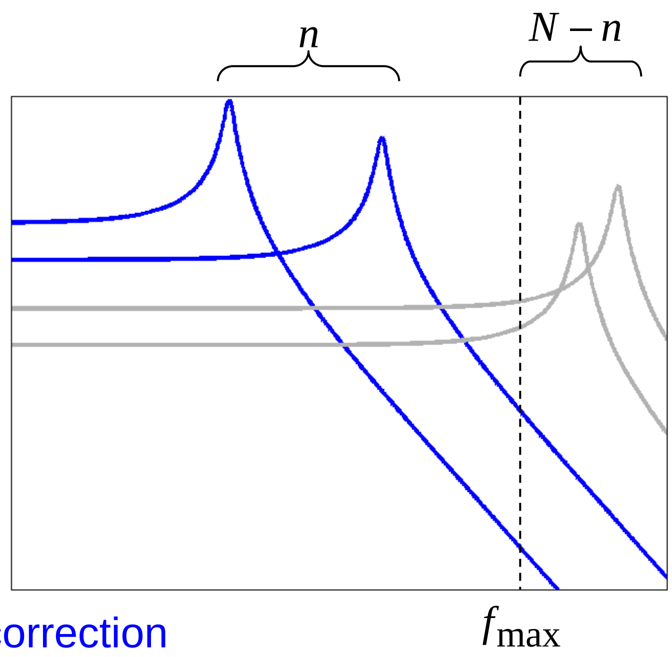

$$X(\omega) = \sum_{k=1}^{n} A_k(\omega)\, \tilde{X}_k$$ no correction

$$X(\omega) = \sum_{k=1}^{n} A_k(\omega)\, \tilde{X}_k + \overline{X}_{res}$$ static correction (mode acceleration)

$$X(\omega) = \sum_{k=1}^{n} A_k(\omega)\, \tilde{X}_k + \overline{X}_{res} + \omega^2\, \overline{Y}_{res}$$ quadratic correction

$$X(\omega) = \sum_{k=1}^{n} A_k(\omega)\, \tilde{X}_k + A_{res}(\omega)\, \tilde{X}_{res}$$ dynamic correction (residual mode)

$$\omega_{res}^2 = X_{res} / Y_{res} = \frac{\sum_{k=n+1}^{N} \tilde{X}_k}{\sum_{k=n+1}^{N} \tilde{X}_k / \omega_k^{\,2}}$$

(weighted harmonic mean of the non-retained eigenvalues)

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FRFs / Example

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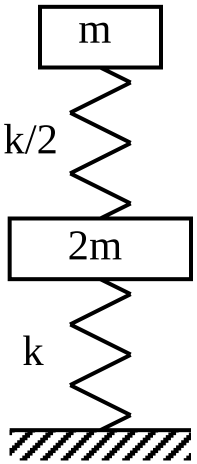
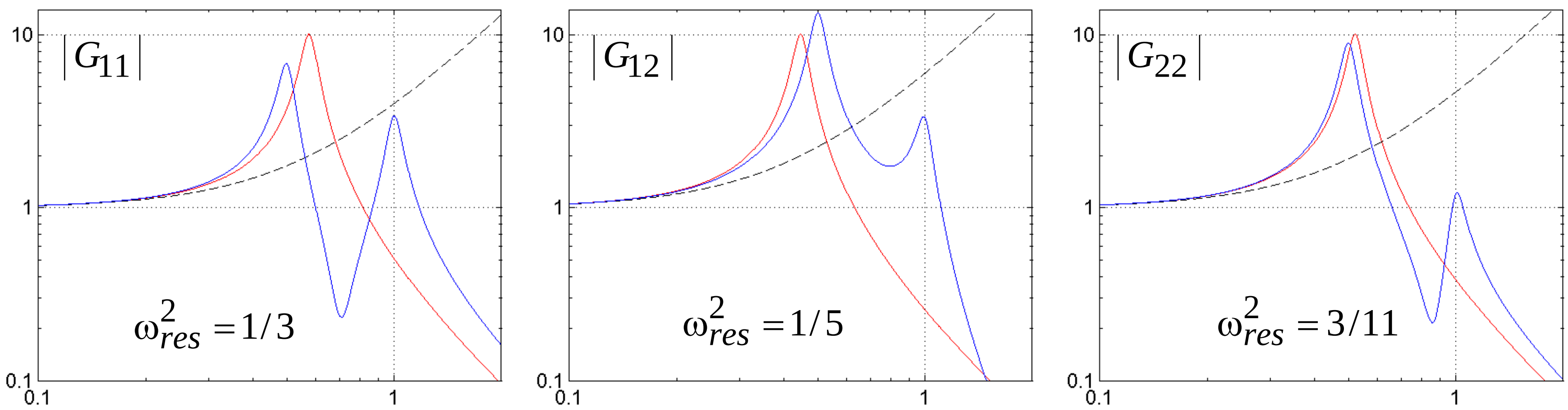
$k = m = 1$



Term	Mode 1	Mode 2
ω_k^2	$1/4$	1
$\tilde{\mathbf{G}}_{ii,\underline{k}}$	$\begin{bmatrix} 2/3 & 4/3 \\ 4/3 & 8/3 \end{bmatrix}$	$\begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 1/3 \end{bmatrix}$
$\tilde{\mathbf{T}}_{ij,\underline{k}}$	$\begin{bmatrix} 2/3 \\ 4/3 \end{bmatrix}$	$\begin{bmatrix} 1/3 \\ -1/3 \end{bmatrix}$
$\tilde{\mathbf{M}}_{jj,\underline{k}}$	$8/3$	$1/3$

$n = 0$ (no retained modes)

- exact response
- dynamic correction
- quadratic correction



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Residual Modes / Introduction

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PROPERTIES

Residual Mode(s)

$$\begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{ik} & \hat{\mathbf{\Phi}}_{ir} & \mathbf{\Psi}_{ij} \\ \mathbf{0}_{jk} & \mathbf{0}_{jr} & \mathbf{I}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{q}_k \\ \mathbf{u}_j \end{bmatrix}$$

- Satisfy orthogonality

$$\begin{cases} \mathbf{\Phi}_{ki}\ \mathbf{M}_{ii}\ \hat{\mathbf{\Phi}}_{ir} = 0 & \forall\ \underline{k}\ ,\ \underline{r} \\ \hat{\mathbf{\Phi}}_{li}\ \mathbf{M}_{ii}\ \hat{\mathbf{\Phi}}_{ir} = 0 & \underline{l} \neq \underline{r} \end{cases}$$

- Are not eigenvectors

$$\left(-\omega_{\underline{r}}^{\ 2}\ \mathbf{M}_{ii} + \mathbf{K}_{ii}\right)\hat{\mathbf{\Phi}}_{i\underline{r}} \neq \mathbf{0}$$

CREATION

⇨ $\mathbf{X}_{ir}$ Starting Ritz vector(s)

⇨ $\hat{\mathbf{X}}_{ir} = \mathbf{X}_{ir} - \mathbf{\Phi}_{ik}\left(\mathbf{m}_{kk}^{-1}\ \mathbf{\Phi}_{ki}\ \mathbf{M}_{ii}\ \mathbf{X}_{ir}\right)$ Orthogonality to normal modes

⇨ $\hat{\mathbf{\Phi}}_{ir} = \hat{\mathbf{X}}_{ir}\ \mathbf{V}_{rr}$ Orthogonality to each other

$$\left(\hat{\mathbf{K}}_{rr} - \hat{\omega}_{\underline{r}}^{2}\ \hat{\mathbf{M}}_{rr}\right)\mathbf{V}_{r\underline{r}} = 0$$

$$\hat{\mathbf{K}}_{rr} = \hat{\mathbf{X}}_{ri}\ \mathbf{K}_{ii}\ \hat{\mathbf{X}}_{ir} \qquad \hat{\mathbf{M}}_{rr} = \hat{\mathbf{X}}_{ri}\ \mathbf{M}_{ii}\ \hat{\mathbf{X}}_{ir}$$

Residual Modes / Excitation

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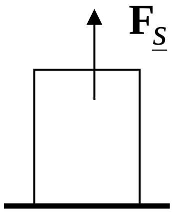
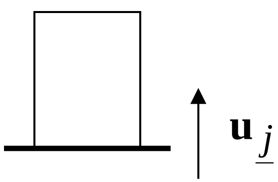
APPLIED FORCE



$$\hat{\mathbf{\Phi}}_{i\underline{s}} = \mathbf{G}_{i\underline{s},res} \quad \Rightarrow \quad \omega_{res}^2 = \frac{\mathbf{G}_{\underline{s}\underline{s},res}}{\mathbf{G}_{\underline{s}i}\,\mathbf{M}_{ii}\,\mathbf{G}_{i\underline{s}}} \qquad \mathbf{G}_{\underline{s}\underline{s}}(\omega)$$

(residual flexibility) (related FRF)

IMPOSED BASE MOTION



$\hat{\mathbf{\Phi}}_{i\underline{j}}$	ω_{res}^2	Related FRF
$\mathbf{\Psi}_{i\underline{j},res}$	$\dfrac{(\hat{\mathbf{\Psi}}_{\underline{j}i}\,\mathbf{K}_{ii}\,\hat{\mathbf{\Psi}}_{i\underline{j}})_{res}}{(\hat{\mathbf{\Psi}}_{\underline{j}i}\,\mathbf{M}_{ii}\,\hat{\mathbf{\Psi}}_{i\underline{j}})_{res}}$	$-\omega^2\,\mathbf{M}_{\underline{j}\underline{j}}(\omega)$
$\sqrt{\mathbf{G}_{ii}}\,\sqrt{\mathbf{M}_{ii}}\,\mathbf{\Psi}_{i\underline{j},res}$	$\dfrac{(\hat{\mathbf{\Psi}}_{\underline{j}i}\,\mathbf{M}_{ii}\,\hat{\mathbf{\Psi}}_{i\underline{j}})_{res}}{(\hat{\mathbf{\Psi}}_{\underline{j}i}\,\mathbf{M}_{ii}\,\mathbf{G}_{ii}\,\mathbf{M}_{ii}\,\hat{\mathbf{\Psi}}_{i\underline{j}})_{res}}$	$\mathbf{M}_{\underline{j}\underline{j}}(\omega)$
$\mathbf{G}_{ii}\,\mathbf{M}_{ii}\,\mathbf{\Psi}_{i\underline{j},res}$	$\dfrac{(\hat{\mathbf{\Psi}}_{\underline{j}i}\,\mathbf{M}_{ii}\,\mathbf{G}_{ii}\,\mathbf{M}_{ii}\,\hat{\mathbf{\Psi}}_{i\underline{j}})_{res}}{(\hat{\mathbf{\Psi}}_{\underline{j}i}\,\mathbf{M}_{ii}\,\mathbf{G}_{ii}\,\mathbf{M}_{ii}\,\mathbf{G}_{ii}\,\mathbf{M}_{ii}\,\hat{\mathbf{\Psi}}_{i\underline{j}})_{res}}$	$\dfrac{\mathbf{M}_{\underline{j}\underline{j}}(\omega)}{-\omega^2}$

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Residual Modes / Example

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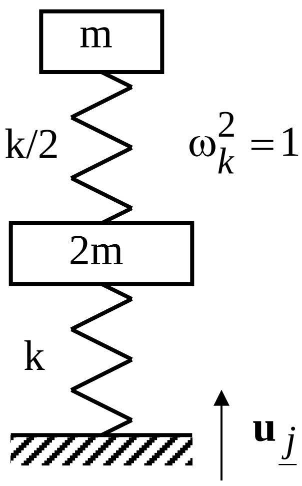
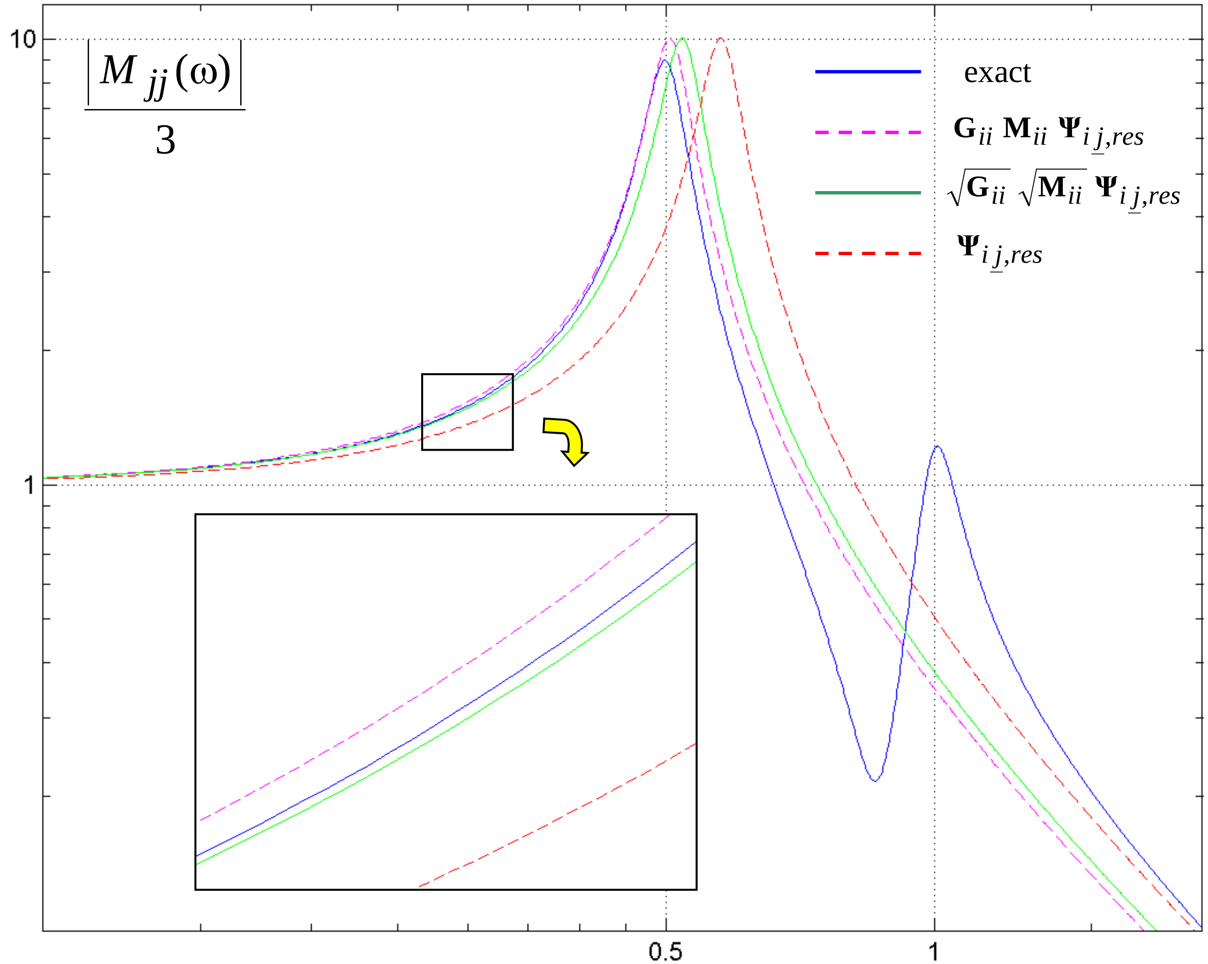
k = m = 1



$\omega_k^2 = 1/4\,,1$

$\hat{\mathbf{\Phi}}_{i\underline{j}}$	ω_{res}^2
$\mathbf{\Psi}_{i\underline{j},res}$	$1/3$
$\sqrt{\mathbf{G}_{ii}}\,\sqrt{\mathbf{M}_{ii}}\,\mathbf{\Psi}_{i\underline{j},res}$	$3/11$
$\mathbf{G}_{ii}\,\mathbf{M}_{ii}\,\mathbf{\Psi}_{i\underline{j},res}$	$11/43$



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Residual Modes / Remarks

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- **Exact only for drive-point FRFs**
- **Approximate for all other FRFs**
- **Easy to compute for dynamic flexibilities** $\hat{\mathbf{\Phi}}_{i\underline{s}} = \mathbf{G}_{i\underline{s},res}$
- **Difficult to compute for dynamic masses** $\hat{\mathbf{\Phi}}_{i\underline{j}} = \sqrt{\mathbf{G}_{ii}}\,\sqrt{\mathbf{M}_{ii}}\,\mathbf{\Psi}_{i\underline{j},res}$

Therefore :

- **Use modal effective parameters for single point excitation**
- **Use residual modes for more complex (distributed) excitation**

Industrial Applications / Modal Identification

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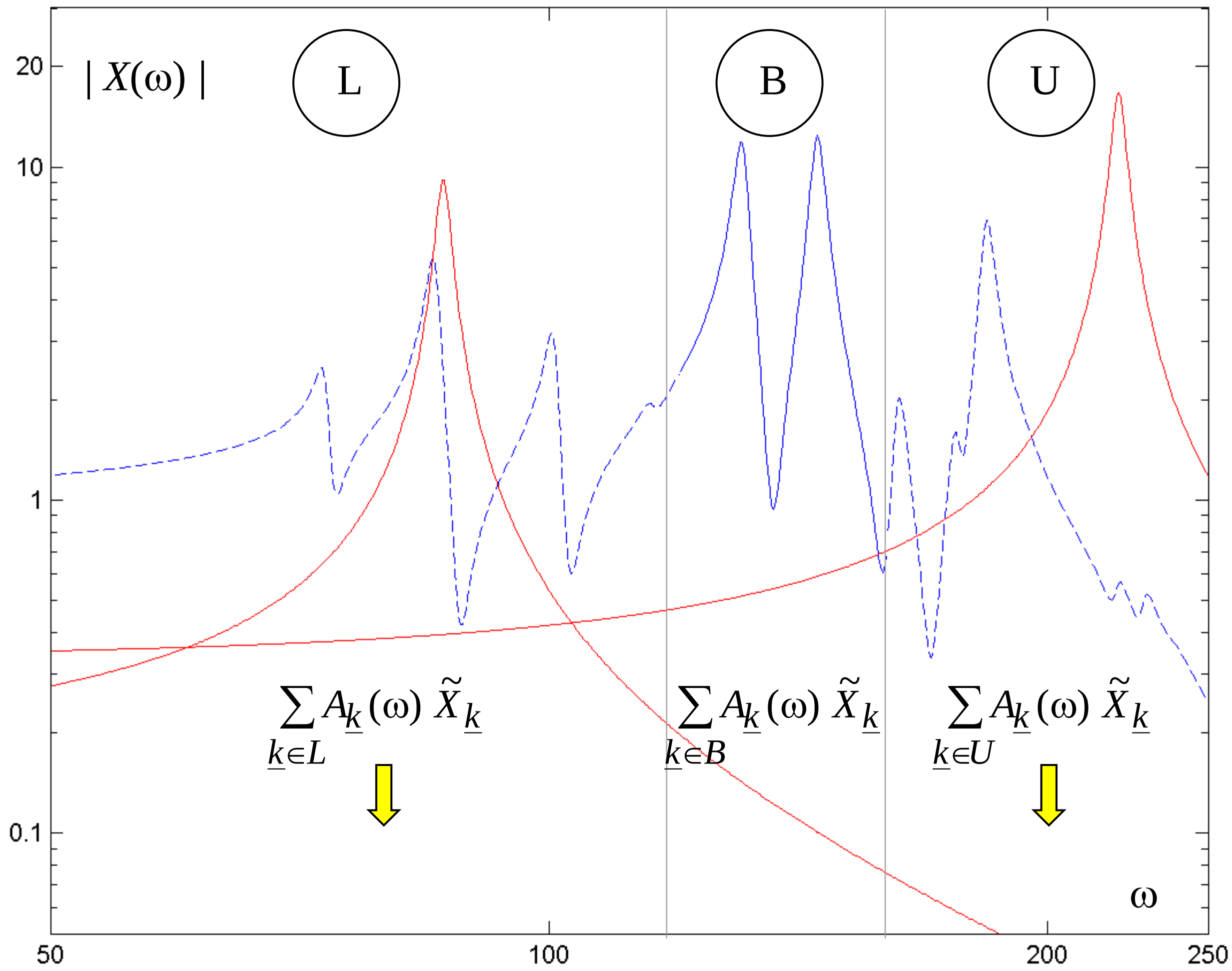
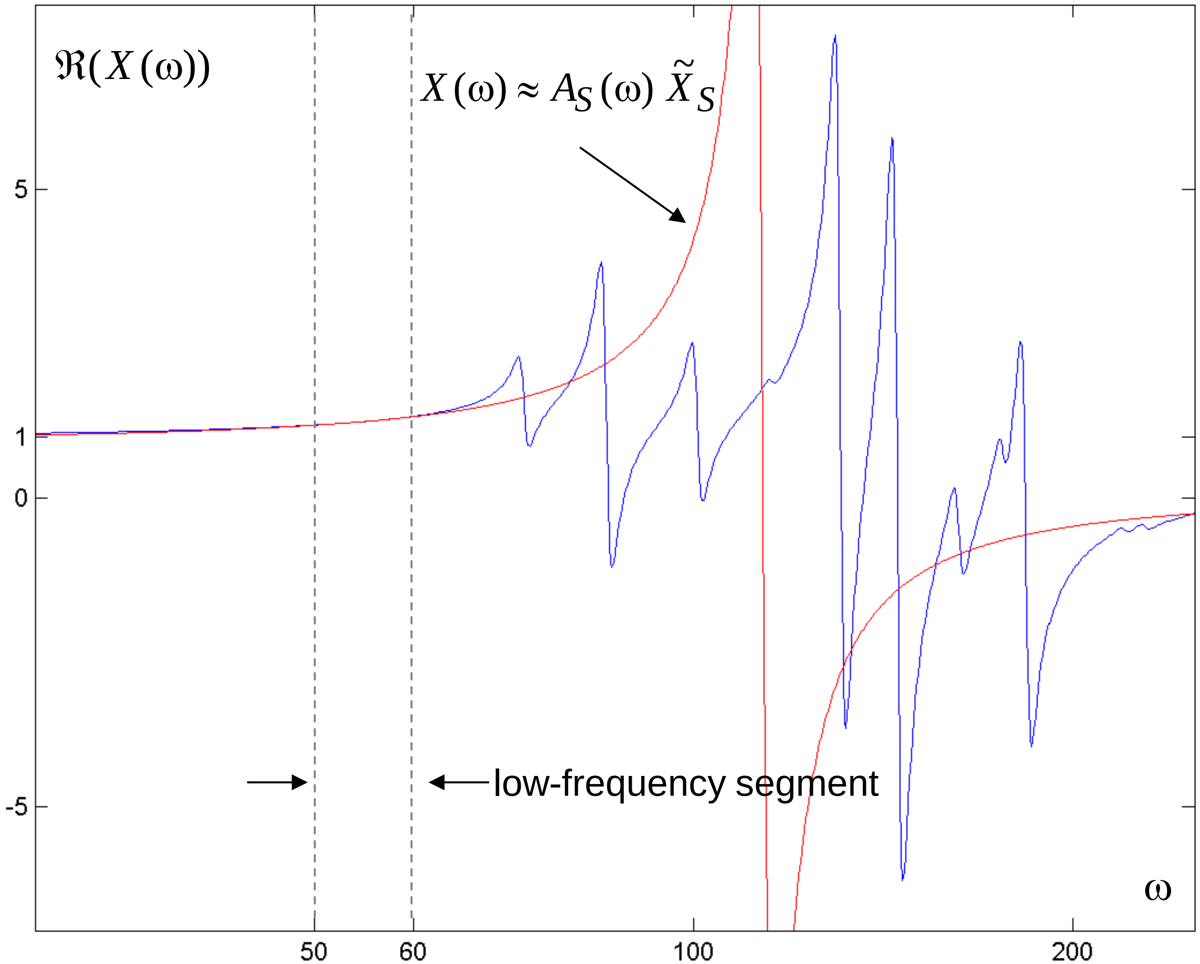


Industrial Applications / Modal Identification



$\tilde{X}_S$ → extrapolated static response

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Industrial Applications / Modal Identification

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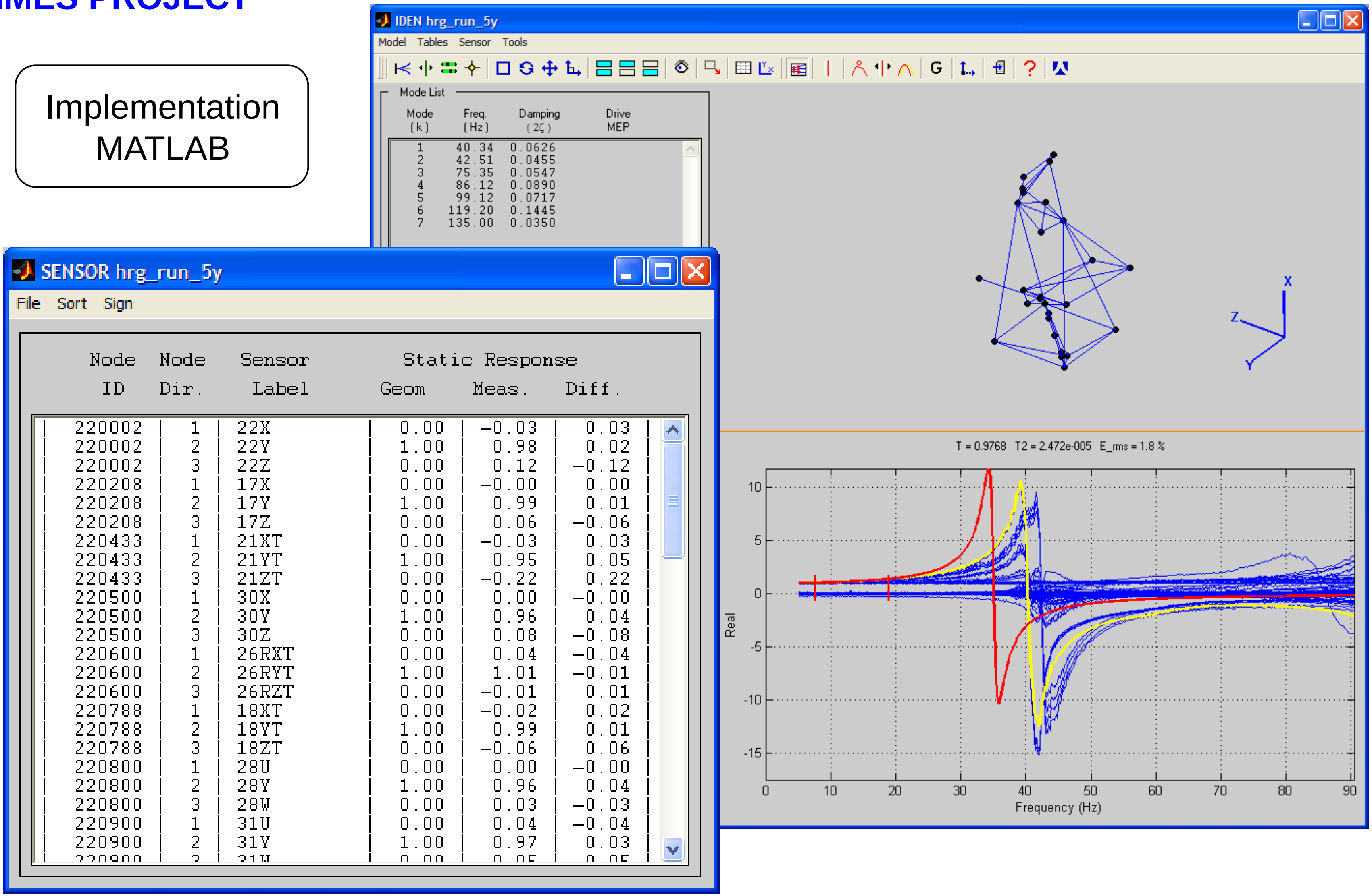
IMES PROJECT

Implementation
MATLAB



Mode (k)	Freq. (Hz)	Damping (2ζ)	Drive MEP
1	40.34	0.0626	
2	42.51	0.0455	
3	75.35	0.0547	
4	86.12	0.0890	
5	99.12	0.0717	
6	119.20	0.1445	
7	135.00	0.0350	

Node ID	Node Dir.	Sensor Label	Static Response Geom	Meas.	Diff.
220002	1	22X	0.00	-0.03	0.03
220002	2	22Y	1.00	0.98	0.02
220002	3	22Z	0.00	0.12	-0.12
220208	1	17X	0.00	-0.00	0.00
220208	2	17Y	1.00	0.99	0.01
220208	3	17Z	0.00	0.06	-0.06
220433	1	21XT	0.00	-0.03	0.03
220433	2	21YT	1.00	0.95	0.05
220433	3	21ZT	0.00	-0.22	0.22
220500	1	30X	0.00	0.00	-0.00
220500	2	30Y	1.00	0.96	0.04
220500	3	30Z	0.00	0.08	-0.08
220600	1	26RXT	0.00	0.04	-0.04
220600	2	26RYT	1.00	1.01	-0.01
220600	3	26RZT	0.00	-0.01	0.01
220788	1	18XT	0.00	-0.02	0.02
220788	2	18YT	1.00	0.99	0.01
220788	3	18ZT	0.00	-0.06	0.06
220800	1	28U	0.00	0.00	-0.00
220800	2	28Y	1.00	0.96	0.04
220800	3	28W	0.00	0.03	-0.03
220900	1	31U	0.00	0.04	-0.04
220900	2	31Y	1.00	0.97	0.03

Industrial Applications / Localized Damping

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Highly damped joints, seals, viscoelastic layers modeled as complex stiffness

- **Dissipative forces poorly represented by retained normal modes**
  - **Need to enrich basis by suitable residual modes**

- $\left(-\omega_{\underline{k}}^{2}\,\mathbf{M}_{ii} + \Re(\mathbf{K}_{ii})\right)\mathbf{\Phi}_{i\underline{k}} = 0$ Retained normal modes
- $\mathbf{F}_{ik} = \Im(\mathbf{K}_{ii})\,\mathbf{\Phi}_{ik}$ Dissipative forces
- $\Re(\mathbf{K}_{ii})\,\mathbf{X}_{ik} = \mathbf{F}_{ik}$ Static modes
- $\mathbf{B}_{ib} = \left[\,\mathbf{\Phi}_{ik} \quad \hat{\mathbf{\Phi}}_{ik}\,\right]$ Enriched basis (real, double in size)

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Industrial Applications / Localized Damping

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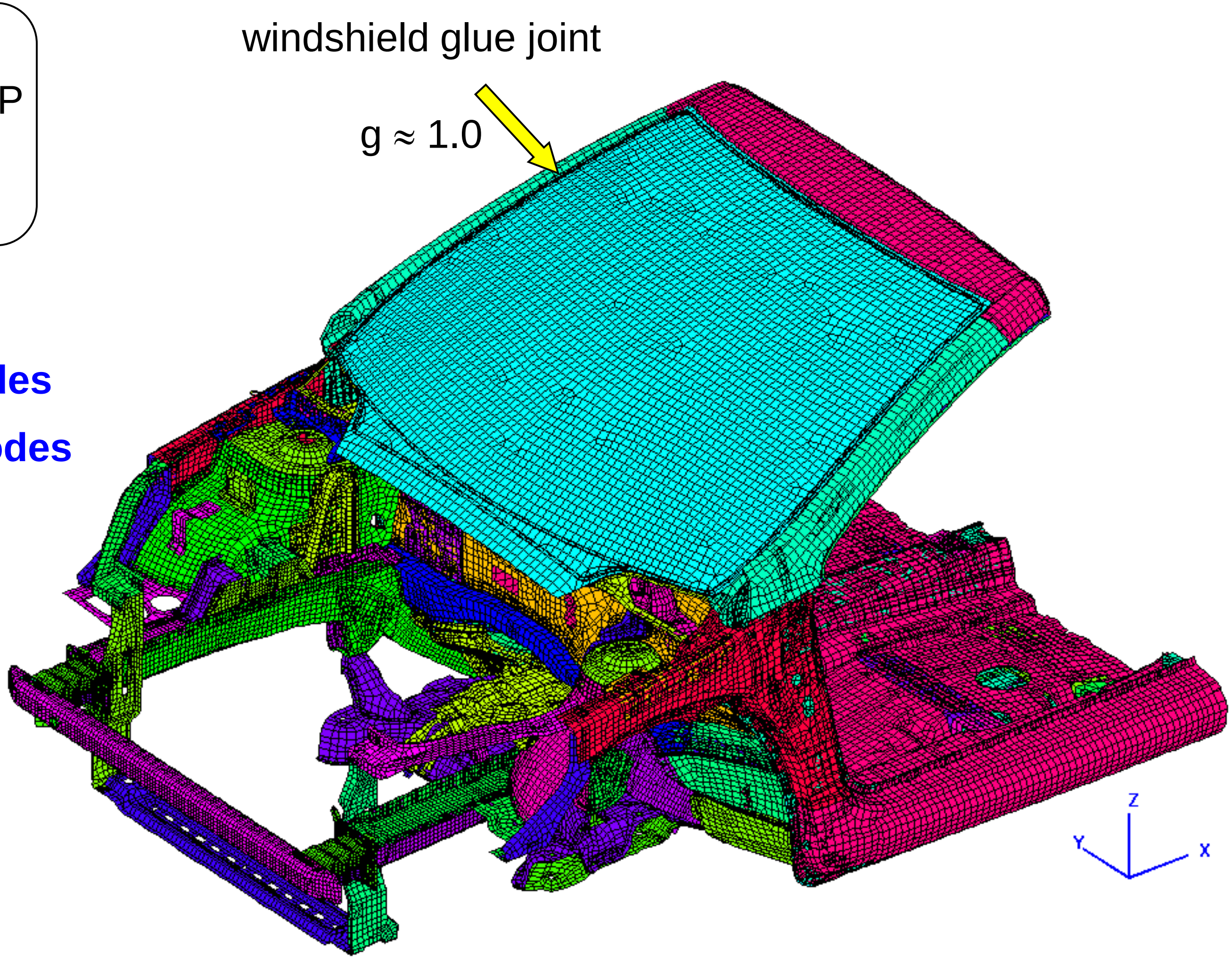
Implementation
NASTRAN / DMAP
- SOL 110
- SOL 111

- **20 normal modes**
- **20 residual modes**



Car Body with Windshield

Industrial Applications / Localized Damping

Equivalent Undamped Natural Frequencies and Modal Damping Factors

Elastic Mode	Nat. Frequency		Modal Damping	
	f_k	Error (%)	g_k	Error (%)
1	□□.48	0.03	2.29	7.08
2	□□.57	0.11	3.90	23.73
3	□□.34	0.56	6.77	26.85
4	□□.41	0.08	2.33	7.27
5	□□.78	0.14	2.93	13.76
6	□□.35	0.01	2.03	0.76
7	□□.16	0.00	2.05	1.01
8	□□.73	0.04	2.29	6.18
9	□□.00	0.00	2.02	0.35
10	□□.05	0.00	2.02	0.48

No Residual Modes

Elastic Mode	Nat. Frequency		Modal Damping	
	f_k	Error (%)	g_k	Error (%)
1	□□.49	0.00	2.14	0.21
2	□□.60	0.00	3.15	0.37
3	□□.52	0.01	5.34	1.05
4	□□.44	0.00	2.17	0.17
5	□□.85	0.01	2.58	0.79
6	□□.35	0.00	2.01	0.01
7	□□.16	0.00	2.03	0.03
8	□□.75	0.00	2.16	0.27
9	□□.00	0.00	2.01	0.02
10	□□.05	0.00	2.01	0.02

Using Residual Modes

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Industrial Applications / Fluid Cavities

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Equations of Motion

$$\left(-\omega^2\begin{bmatrix}\mathbf{M}_{ss} & \mathbf{0}_{sf}\\ \mathbf{C}_{fs} & \mathbf{M}_{ff}\end{bmatrix}+\begin{bmatrix}\mathbf{K}_{ss} & -\mathbf{C}_{sf}\\ \mathbf{0}_{fs} & \mathbf{K}_{ff}\end{bmatrix}\right)\begin{bmatrix}\mathbf{u}_s\\ \mathbf{p}_f\end{bmatrix}=\begin{bmatrix}\mathbf{F}_s\\ \dot{\mathbf{Q}}_f\end{bmatrix}$$

$\mathbf{M}_{ss}$	$\mathbf{K}_{ss}$	Structure Mass and Stiffness Matrices
$\mathbf{u}_s$	$\mathbf{F}_s$	Structure Displacement and Force
$\mathbf{M}_{ff}$	$\mathbf{K}_{ff}$	Fluid "Mass" and "Stiffness" Matrices
$\mathbf{p}_f$	$\dot{\mathbf{Q}}_f$	Fluide pressure and Acoustic Source
$\mathbf{C}_{fs}$		Structure-Fluid Coupling Matrix

Modal Projection using uncoupled modes

$$\mathbf{u}_s=\mathbf{\Phi}_{sn}\,\mathbf{q}_n \qquad \left(-\omega_{\underline{n}}^2\mathbf{M}_{ss}+\mathbf{K}_{ss}\right)\mathbf{\Phi}_{i\underline{n}}=0$$

$$\mathbf{p}_f=\mathbf{\Phi}_{fm}\,\mathbf{q}_m \qquad \left(-\omega_{\underline{m}}^2\mathbf{M}_{ff}+\mathbf{K}_{ff}\right)\mathbf{\Phi}_{i\underline{m}}=0$$

} poor convergence because modes neglect coupling

Industrial Applications / Fluid Cavities

ENRICHED BASIS

	structure	fluid
normal modes	$\left(-\omega_{\underline{n}}^2 \mathbf{M}_{ss} + \mathbf{K}_{ss}\right)\mathbf{\Phi}_{s\underline{n}} = \mathbf{0}_s$	$\left(-\omega_{\underline{m}}^2 \mathbf{M}_{ff} + \mathbf{K}_{ff}\right)\mathbf{\Phi}_{f\underline{m}} = \mathbf{0}_f$
static vectors	$\mathbf{K}_{ss}\mathbf{X}_{sm} = \mathbf{C}_{sf}\mathbf{\Phi}_{fm}$	$\mathbf{K}_{ff}\mathbf{X}_{fn} = \mathbf{C}_{fs}\mathbf{\Phi}_{sn}$
enriched basis	$\mathbf{\Phi}_{sb} = \left[\mathbf{\Phi}_{sn} \quad \hat{\mathbf{\Phi}}_{sm}\right]$	$\mathbf{\Phi}_{fb} = \left[\mathbf{\Phi}_{fm} \quad \hat{\mathbf{\Phi}}_{fn}\right]$

(residual modes)

Industrial Applications / Fluid Cavities

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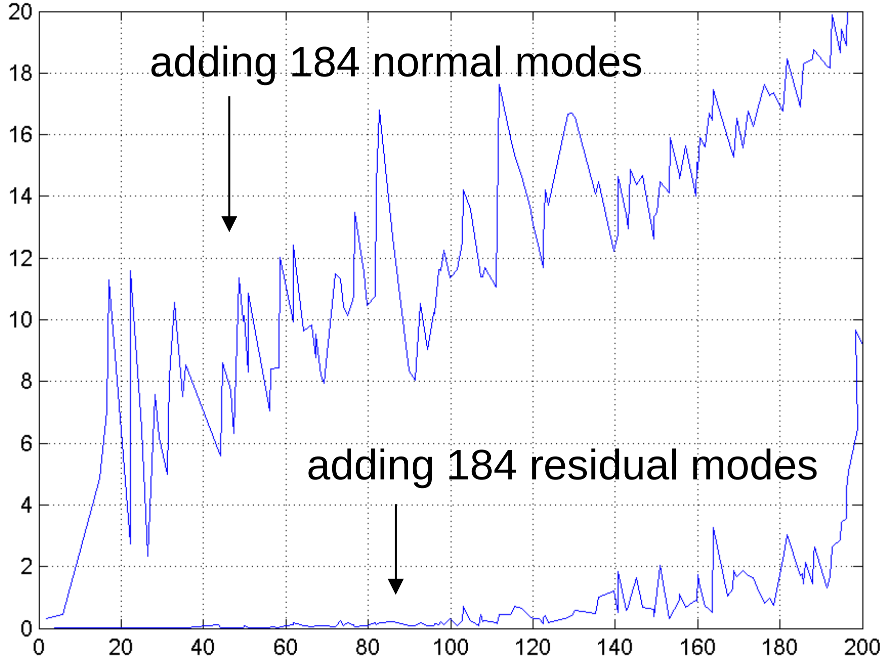
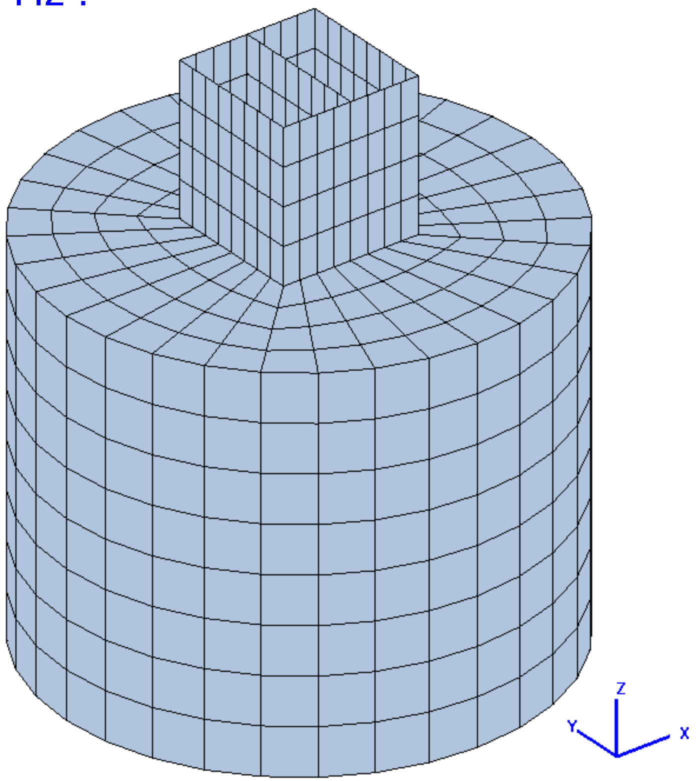
Uncoupled modes up to 300 Hz :

- 152 structure (n)
- 32 fluid (m)
- 184 total ($n + m$)

Error in Natural Frequencies (%)



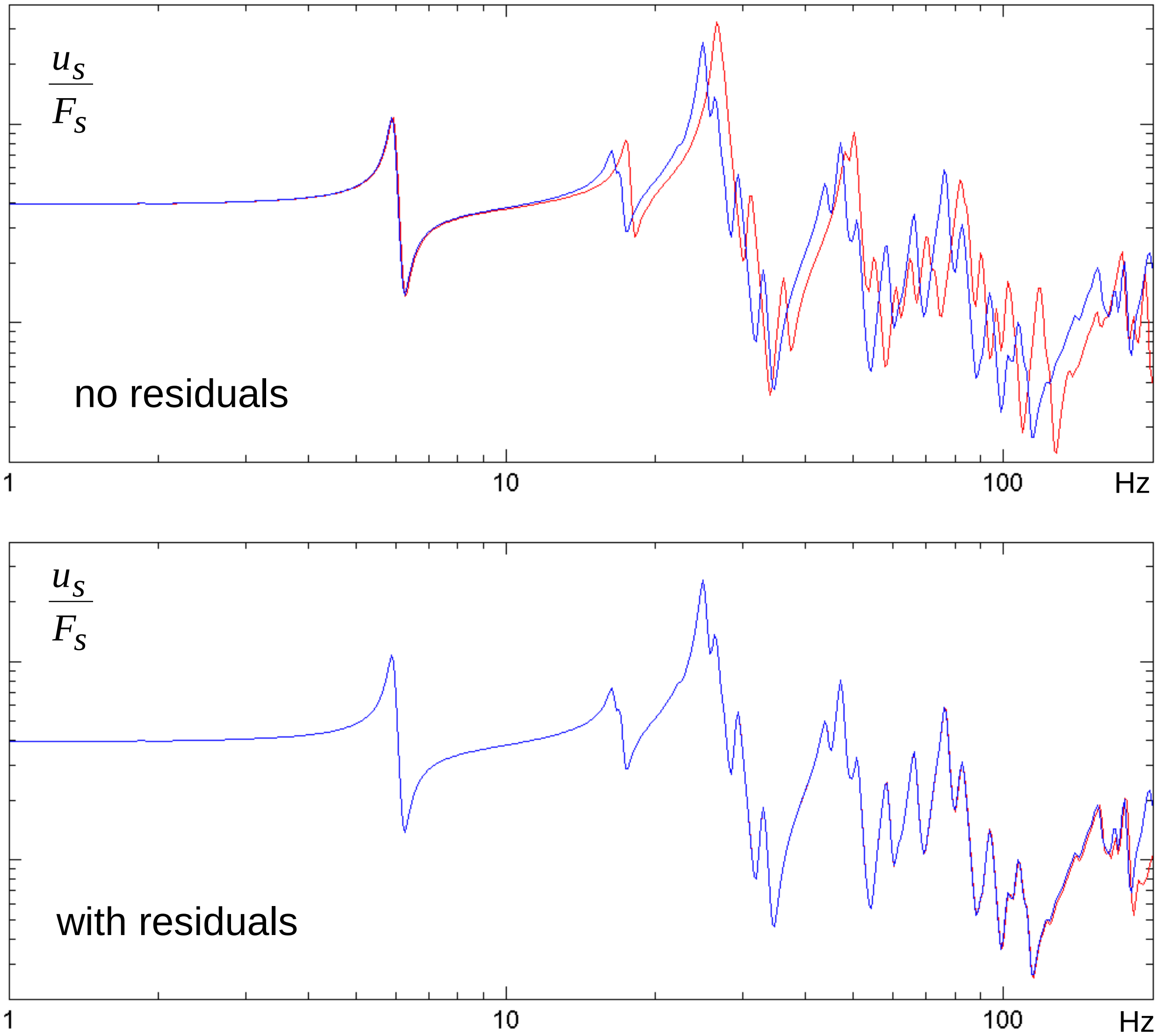


Marine Ballast with 2 Cavities

Industrial Applications / Fluid Cavities

Top Modal

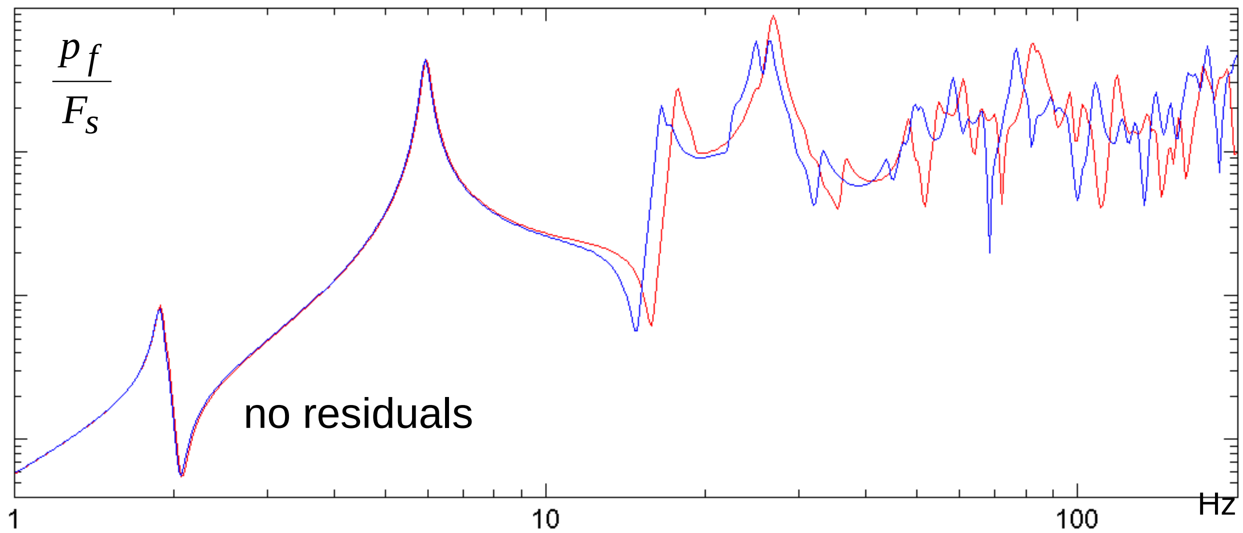
displacement response

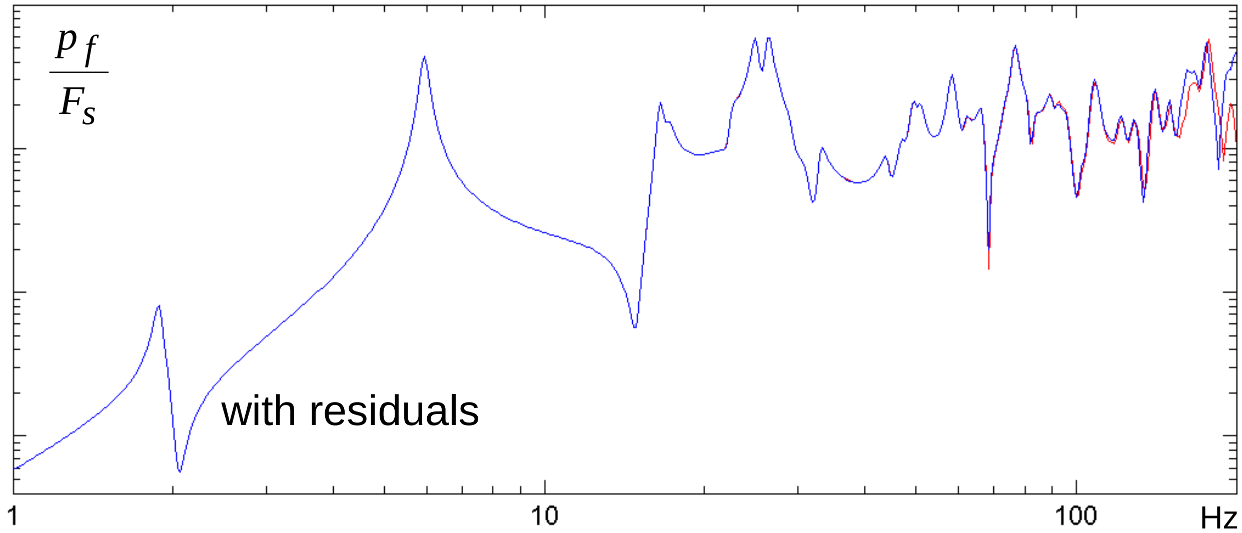


Industrial Applications / Fluid Cavities

Top Modal

pressure response





Conclusions

- Residual Modes using MEPs or from Ritz Vectors
- Efficiency and Simplicity of Mode Superposition Techniques
- Easy to implement in Standard FE Codes
- Ongoing Development : Modal Synthesis / Reduced Models

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