Vibro-acoustic Response to Diffuse Acoustic Field Using a Modal Approach and Application to the IXV Re-entry Vehicle Design Justification

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Abstract
Diffuse acoustic fields are typically created in spaces with many reflecting surfaces and small sound absorption. They are commonly used in the space industry to describe the acoustic environment inside the launcher's fairing. This acoustic environment can be critical to payload structures with large exposed surfaces and high area-to-mass aspect ratios. Its impact on structural integrity has to be evaluated at the earlier stages of design. Numerical methods to represent the diffuse sound field excitation and predict the structural vibro-acoustic response have been developed in the past years. The most detailed solution is a BEM/FEM (Boundary Element Method and Finite Element Method) approach that combines a fluid domain model (BEM) and a structural model (FEM) to solve the fully coupled vibro-acoustic problem. Several commercial software packages offer this BEM/FEM approach. However this solution can require considerable pre-processing time as well as substantial computational resources and may not be adapted to preliminary design justification.
The main focus of this work is to present an alternative pre-processing tool developed in collaboration with TOP MODAL to perform the diffuse vibro-acoustic response in standard FEM MSC.Nastran code. The motivation was the development of a pre-processing tool compatible with large FE models that handles diffuse field spatial-temporal correlation and enables the use of MSC.Nastran/Patran computational and post-processing capabilities. An industrial application is included using the IXV (Intermediate eXperimental Vehicle) spacecraft Windward and Nose Assembly design justification. The IXV is a European Atmospheric Re-entry Demonstrator developed with Thales Alenia Space Italy as prime contractor and funded by ESA. The IXV Windward and Nose Assembly are lightweight CMC (Ceramic Matrix Composite) structures manufactured by Snecma Propulsion Solide to ensure thermal protection during the spacecraft atmosphere re-entry. These structures are exposed to the diffuse sound field environment inside the fairing during launcher lift-off.

1. Introduction

Simulation of structural response of structures under acoustic loadings has received a lot of attention from aerospace industrials and researchers in the past years. The constant effort of mass reduction and structural optimization with the use of composite materials has led to the design of a growing number of lightweight structures with high area-to-mass aspect ratios. Such structures are likely to be subjected to high levels of vibrations and stresses when exposed to acoustic loads. During the past years, Snecma Propulsion Solide (SPS) has increased the development and manufacturing of CMC (Ceramic Matrix Composite) structures with high thermo-structural capabilities for the aerospace industry. In some applications, the CMC structures can be subjected to severe acoustic environments such as acoustic noise at launcher's lift-off or turbofan sonic fatigue for aircraft applications. Efforts have been made in SPS simulation department to develop FE-based simulation methods able to perform the structural response to these environments.

The objective of this paper is to present the pre-processing tool developed in collaboration with TOP MODAL to perform the vibro-acoustic response in MSC.Nastran. MSC.Nastran is the standard FEA code used at SPS for dynamic analysis. Response to acoustic pressure load can typically be carried out in MSC.Nastran as a random response analysis. However the user can only define acoustic loadings as correlated pressure fields. In many applications, a specific spatial-temporal correlation function has to be taken into account.

For example a diffuse sound field correlation is commonly used in the space industry to describe the environment inside the fairing. Turbulent boundary layer correlation models may also be used for aircraft applications. Detailed solutions are usually provided by commercial software using a BEM/FEM approach that combines a fluid domain model (BEM) and a structural model (FEM) to solve the fully coupled vibro-acoustic problem. Acoustic sources included in the fluid domain are used to simulate a physical acoustic pressure field. A diffuse sound field is thus simulated by including a large number of plane wave acoustic sources in the fluid domain. However this approach requires pre-processing time and computational resources that may be not adapted to preliminary design justification. In addition, FEM/BEM software was not available in the department’s current FEA solutions package and would require investment and staff training.
This motivated the development of an alternative pre-processing tool for MSC.Nastran that handles specific spatial-temporal correlation function for the vibro-acoustic problem. The method presented in this work will consider a diffuse sound field correlation. But the tool can be straightforwardly generalized to any kind of correlation described by a mathematical function or test data. Special attention was also given to enable the use of MSC.Patran post-processing capabilities such as RMS contour plots.

The idea is to condense the physical system using the normal modes of the structure in order to obtain a reduced but equivalent system expressed entirely in modal coordinates. Generalized modal forces are computed taking into account the diffuse field correlation and are commonly referred to as joint-acceptance terms. The method is applicable to low acoustic coupling situations where acoustic radiation can be neglected. This limitation is a reasonable simplification within the range of applications for CMC structures developed by SPS.
The condensation procedure by modal approach offers great benefits in terms of computational time and resources while being able to accurately represent physics of the problem. Application examples on simply supported beam and plate are provided to illustrate and validate the formulation and tool features. The importance of taking into account the diffuse field correlation is highlighted by comparison with results using a correlated acoustic field.
An industrial application is included using the IXV (Intermediate eXperimental Vehicle) spacecraft Windward and Nose Assembly design justification. The IXV is a European Atmospheric Re-entry Demonstrator developed with Thales Alenia Space Italy as prime contractor and funded by ESA. The IXV Windward and Nose Assembly are lightweight CMC (Ceramic Matrix Composite) structures manufactured by SPS to ensure thermal protection during the spacecraft atmosphere re-entry. These structures are exposed to the diffuse sound field environment inside the fairing during launcher lift-off. Assessment of the structural response to this specific acoustic environment has to be integrated in the design justification procedure.

2. Formulation of the method

2.1 Equivalent random excitation

Diffuse sound field model is widely used to represent the acoustic pressure in enclosures with many reflection surfaces and small sound absorption. It is commonly used in the space industry to describe the acoustic environment inside the launcher’s fairing. The very high-amplitude combustion noise generated by first-stage main engine at lift-off reflects upward from the ground and envelopes the whole launcher. Vibrations of the fairing structure under this intense lift-off noise are transmitted to the air enclosed in the cavity and a diffuse acoustic field is generated by multiple sound reflections and diffractions on exposed surfaces as illustrated on Figure 1.

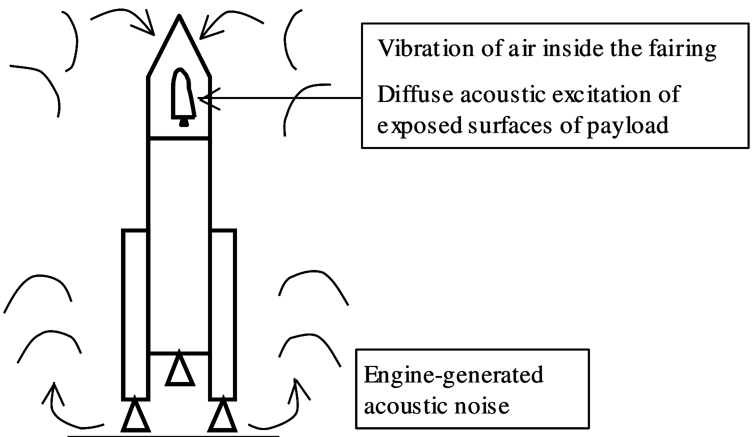


Figure 1 : Payload acoustic environment at lift-oft

It has been shown and tested experimentally (first in 1955 by Cook and al.[1], [2]) that the spatial and temporal correlation of a diffuse pure-tone sound field behaves as a sin(*kr*)/*kr* function (cardinal sine) for waves arriving from all directions, with *k* being the wave number and *r* the distance between two observation points.

Typical requirements for payloads subjected to the launcher's fairing acoustic loads are provided as Sound Pressure Levels N (SPL) in dB per octave. The total Power Spectral Density (PSD) $\mathbf{S}_{pp}(\omega)$ associated with the incident acoustic wave is assumed constant over the octave bandwidth as is given by the following expression :

$$S_{pp}(\omega) = \frac{P_{ref}^2 \, 10^{(N/10)}}{\Delta f} \text{ in Pa²/Hz} \quad \textbf{(1)}$$

With P_{ref} the reference pressure (2×10^{-5}Pa in air), N the SPL in dB and Δf the octave bandwidth in Hz.

Taking into account the diffuse correlation expression, the acoustic pressure field can be described in the frequency domain by cross spectral density functions (2) as the product of a frequency dependent amplitude term $\mathbf{S}_{pp}(\omega)$ defined in (1) and a spatially and frequency dependent phase term f_d (3):

$$p(x_i, x_j \omega) = S_{pp}(\omega) f_d(x_i, x_j, \omega) \quad \textbf{(2)}$$

$$\text{and } f_d(x_i, x_j, \omega) = \frac{\sin(\mathrm{k}|x_i - x_j|)}{\mathrm{k}|x_i - x_j|} \quad \textbf{(3)}$$

Where k = ω/c is the wave number with c the speed of sound in the air. $|x_i - x_j|$ is the distance between two points on the exposed surface of the structure as represented in Figure 2 below.

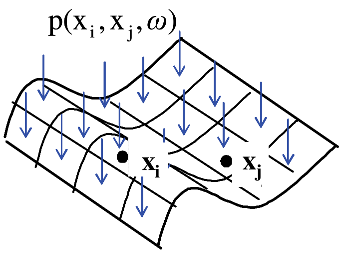


Figure 2 : pressure field on exposed surface

The correlation function f_d is plotted on Figure 3, as a typical cardinal sine function.

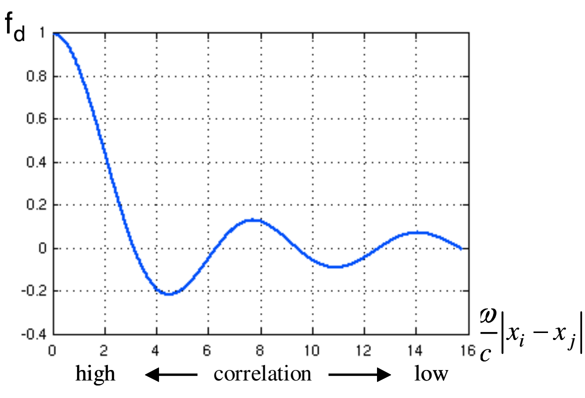


Figure 3 : diffuse acoustic field correlation function

In low frequencies or if the dimensions of the structure are very small compared to the acoustic wavelength λ_{ac} = c/f, the correlation function tends to unity and diffuse excitation is equivalent to a fully correlated acoustic field. In high frequencies or if we consider two points at a distance much larger than the acoustic wave length, the correlation tends to zero and the pressure field is uncorrelated. Maximal vibro-acoustic coupling will be observed if the acoustic wavelength λ_{ac} at a natural frequency corresponds to the spatial wavelength λ_k of the mode. In this situation the diffuse acoustic pressure acting on the structure will be in phase with the modal deformation shape.

Note that the sound pressure amplitude described by $\mathbf{S}_{pp}(\omega)$ is equation (1) is supposed to be spatially constant and that only the so called "blocked" pressure is considered and radiated pressure neglected.

For the non-coupled solution, and as suggested by Figure 2, the diffuse acoustic pressure input on a FE discretised surface can be assimilated to the action of equivalent random nodal excitation forces normal to the surface. Using relation (2) and considering an equivalent nodal area A_i (see Figure 4), the expression of nodal forces in a PSD matrix for NE wetted nodes is given below:

$$\mathbf{S}_{F_i F_i}(\omega) = (\mathbf{A}_{ii}\ f_{d\,ii}(\omega)\ \mathbf{A}_{ii})\,\mathbf{S}_{pp}(\omega) \tag{4}$$

$\mathbf{S}_{F_i F_i}(\omega)$: matrix of excitation PSD forces relative to wetted nodes (real, symmetric, NE×NE)

$\mathbf{A}_{ii}$: Diagonal matrix of nodal areas

$f_{d\,ii}(\omega)$: spatial correlation matrix with terms given by expression (3)

$\mathbf{S}_{pp}(\omega)$: acoustic pressure PSD amplitude given by expression (1)

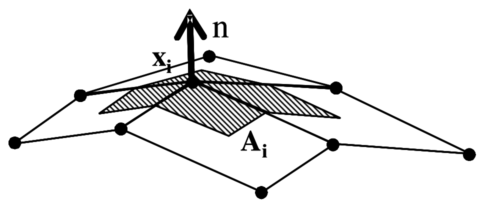


Figure 4 : equivalent nodal area

2.2 Modal approach for the vibro-acoustic response

Using expression (4) with diffuse acoustic pressure input derived as a full matrix of PSD excitation forces, the PSD response $\mathbf{S}_{uiui}$ can be calculated as a standard random response by multiplication with transfer function matrices:

$$\mathbf{S}_{u_i u_i}(\omega) = \mathbf{H}^*_{u_i F_i}(\omega)\,\mathbf{S}_{F_i F_i}(\omega)\,\mathbf{H}_{F_i u_i}(\omega) \tag{5}$$

$\mathbf{H}_{u_i F_i}(\omega)$: transfer function between excitation forces and response

$\mathbf{S}_{F_i F_i}(\omega)$: matrix of excitation PSD forces relative to wetted nodes

u_i : response quantity (stress, acceleration, etc…)

At this point of the formulation, implementation of relation (5) could be possible in MSC.Nastran (or any other FEA code), as a random analysis frequency response performed by direct method or modal superposition. It would require a pre-processor to compute and write in Nastran format all PSDs and cross PSDs forces at wetted nodes using relation (4). Relation (5) is then directly handled by Nastran or performed by the use of Patran module MSC.Random for more post-processing capabilities. However this would be quite a tedious and time expensive task, even for relatively small size models. If NE is the number of wetted nodes, NE Nastran load subcases have to be defined to compute the NE $\mathbf{H}_{uiFi}$ transfer functions. For each force loadcases, PSD and cross PSDs have to be written which represents (NE²+NE)/2 Nastran RANDPS and TABRND entries. This would represent more than one million of entries for a model with only 1500 wetted nodes. The code also has to handle this large and full matrix of PSD excitation forces for each N_{FREQ} frequency steps required to catch modal peaks over a frequency range that typically goes up to several kHz for rocket motor acoustic noise. This limits the use of this direct approach to small models that do not exceed about 10² of wetted nodes.

As it is widely used in structural dynamics, a normal mode reduction formulation instead of the direct approach mentioned above appears to be very well suited to solve the vibro-acoustic problem for large models with reasonable computational time and resources.

The idea is to project the excitation forces defined in relation (4) on the normal modes of the structure and work with generalized modal forces $\mathbf{S}_{FkFk}$ as defined below:

$$\mathbf{S}_{F_k F_k}(\omega) = \mathbf{\Phi}_{ki}\,\mathbf{S}_{F_i F_i}(\omega)\,\mathbf{\Phi}_{ik} = \mathbf{\Phi}_{ki}\,[(\mathbf{A}_{ii}\,f_{dii}(\omega)\,\mathbf{A}_{ii})\,\mathbf{S}_{pp}(\omega)]\,\mathbf{\Phi}_{ik} \tag{6}$$

$\mathbf{\Phi}_{ik}$: matrix of normal modes (NM×NM)

The generalized $\mathbf{S}_{FkFk}$ PSD forces take into account the diffuse field correlation function f_{dii} and are commonly referred to as joint-acceptance terms.

The first advantage of the modal projection is the reduction of the problem size from NE wetted nodes to NM number of modes. It is also interesting to note that cross terms in the joint-acceptance PSD matrix appear to be small compared to diagonal terms and may be omitted as we will illustrate further down. This comes from the fact that normal modes are orthogonal and the problem has been somehow “uncoupled” by modal projection. This property is very useful to perform a rapid but most times very accurate calculation of the vibro-acoustic response with a diagonal joint-acceptance matrix as preliminary approach.

2.3 Implementation in MSC.Nastran

A Matlab pre-processing tool was developed to compute the generalized PSD forces from expression (6). Main calculations (modal analysis, transfer functions and multiplication with excitation forces) are performed in Nastran. Transfer functions must be now computed with generalized forces, which is possible in MSC.Nastran by the use of generalized degrees of freedom (dof) defined as SPOINT entries. Nastran MPC entries are used to link generalized SPOINT dof to physical dof. A set *s* of physical dof is selected by QR decomposition for a well conditioned problem. The relations are given by the following expressions:

$$\mathbf{u}_s = \mathbf{\Phi}_{sk}\ \mathbf{q}_k \tag{7}$$

$$\mathbf{q}_k = (\mathbf{\Phi}_{sk})^{-1}\ \mathbf{u}_s \tag{8}$$

MPC relations are written using relation (8), where q_k are associated to generalized SPOINT dof and $\mathbf{\Phi}_{sk}$ is the normal mode matrix relative to the selected set s of physical dof.

Flowchart diagram of the vibro-acoustic response computation is illustrated on Figure 5:

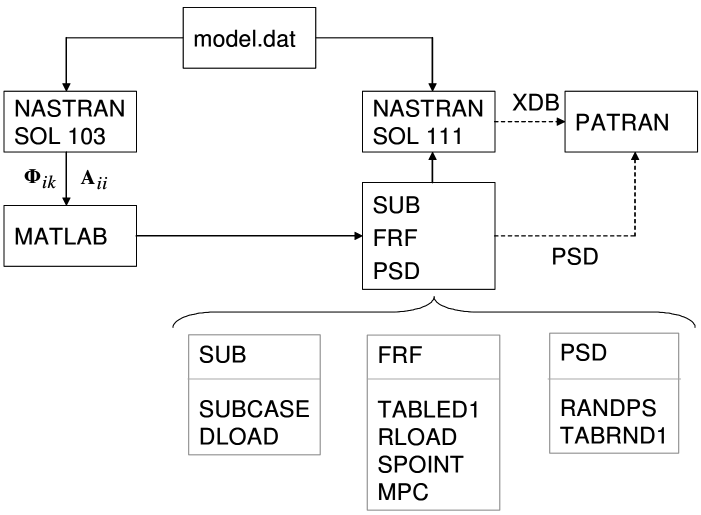


Figure 5 : flowchart diagram of vibro-acoustic response with a matlab pre-processor for Nastran

A normal modes analysis (SOL 103) with unit PLOAD pressure input on the wetted surface is performed to compute and import in Matlab the NM normal eigenvectors $\mathbf{\Phi}_{ik}$ and nodal equivalent areas A_i at wetted nodes. If non-negligible, the effect of added mass of air can be included by means of the virtual mass technique with MFLUID Nastran entries for the normal modes analysis. Generalized forces are calculated and Nastran input files for frequency response analysis are written. The frequency response analysis (SOL 111 or SOL108) is performed to compute modal transfer functions $\mathbf{H}_{uiFk}$. Patran module MSC.Random is used to calculate the random response from the frequency response result file (.xdb) and a "PSD" input file that includes PSD joint-acceptance matrix for the acoustic field correlation (Nastran RANDPS and TABRND1 entries).

The advantage of using MSC.Random is that contour plots of RMS data are straightforward to obtain by using Patran post-processing capabilities. It is also very useful in a sense that once transfer functions have been computed in the .xdb file, the response to any correlation input matrix can be performed by selecting a specific PSD input file without having to re-run the frequency analysis (most costly operation). This way, a quick random response with a fully correlated field (low frequency approximation for diffuse field) or diffuse excitation without cross terms can be performed in a first time and transfer functions remain available for a further analysis with a more detailed PSD matrix input file.

A mode filtering feature has also been added to the Matlab pre-processor. It is very useful in a sense that many modes of the structures may not be excited by the diffuse acoustic field (e.g. local modes). A simple criteria based on the generalized force PSD can be used to remove such modes in the PSD joint-acceptance matrix calculation for a more effective solution. For each mode k the following criterion is computed which can be shown to be an upper bound of $\mathbf{S}_{FkFk}$ over all frequencies (assuming a unit S_{pp}, see equation (6)).

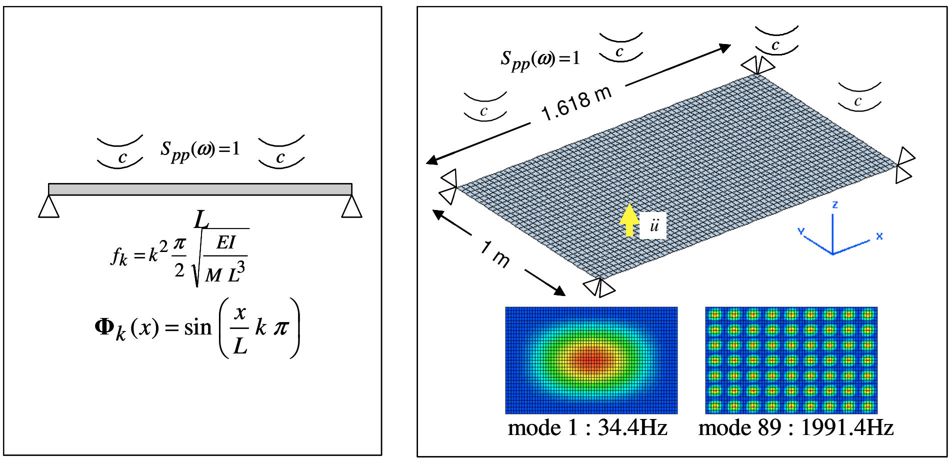
$$C_k = \left(\sum_i A_i \left| \Phi_{ik} \right| \right)^2 \qquad \mathbf{(9)}$$

By normalizing C_k to a maximum of 1, all modes with values of C_k less than a user-defined threshold value (for example 10^{-6}) can be omitted since they are not excitable.

3. Application examples

3.1 Simply supported beam and plate

The methodology and tools features were validated on application examples such as a simply supported beam and plate with some results available in the literature [3,4]. Illustration of the test examples set-up is provided on Figure 6:



Simply supported beam **Simply supported plate**

Figure 6 : simply supported beam and plate under unit diffuse acoustic loading

Joint-acceptance PSD terms $\mathbf{S}_{FkFk}$ are plotted and presented on Figure 7 for the simply supported beam (diagonal 'auto" terms and cross terms). Highest participation at low frequency is demonstrated for low-order odd modes as (1,1) while contribution of high-order even modes (e.g (6,6)) is negligible. As mentioned earlier, participation of cross PSD terms is limited and becomes fairly negligible in high frequency.

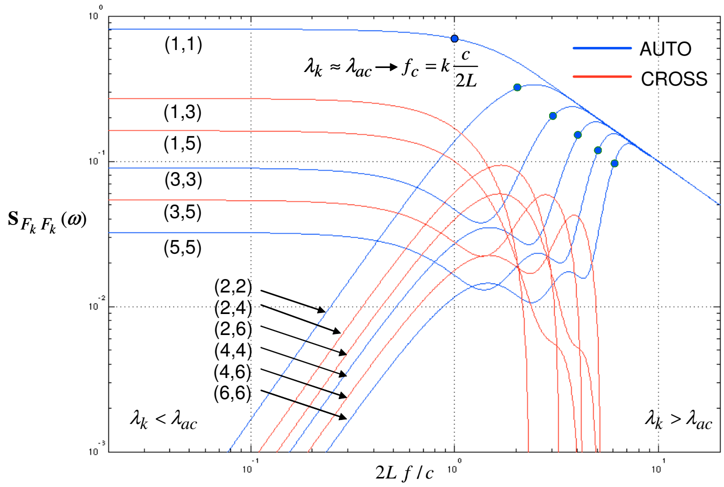


Figure 7 : joint-acceptance terms for simply supported beam

Acceleration PSD response is documented on Figure 8 for the rectangular simply supported plate on four edges (response dof u located at the middle of the quarter plate). PSD response to diffuse acoustic loading is computed for a unit sound pressure level over the [0-2000]Hz frequency range. 89 modes from 34Hz to 2000Hz are included in the numerical model. Response is computed for the complete diffuse joint-acceptance matrix (EXACT), joint-acceptance without cross terms (without XSD) and for a fully correlated sound field.

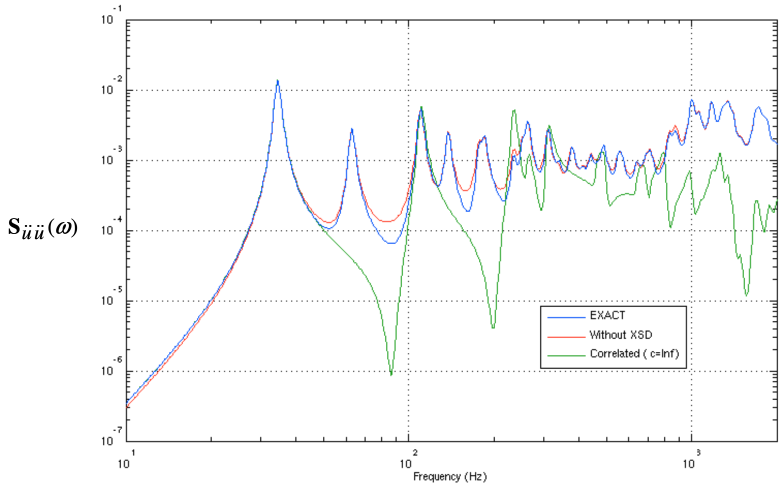


Figure 8 : joint-acceptance terms for simply supported beam

Peaks observed in the PSD response typically correspond to structural natural frequencies of the simply supported plate. In low frequency and for the first peak, the correlated approach is similar to the exact method. However a consequent deviation is illustrated as frequency increases. Higher frequency modes may exhibit significant response to the diffuse correlation while their participation is absent for correlated excitation. It can be noted that the PSD response performed with only diagonal joint-acceptance terms is very similar to the exact solution over the whole frequency range. RMS acceleration results (integration of PSD curves presented on Figure 8) are listed in Table 1:

Table 1: RMS acceleration results for the simply supported plate

	RMS (m/s²)	Relative difference
EXACT	2.26	-
Without XSD	2.28	+ 0.9%
Correlated	0.96	-57.5%

For the plate example, Table 1 illustrates the necessity to take into account the diffuse field correlation in comparison with correlated field results. It is also demonstrated that cross correlation terms can be omitted for a less resource-expensive analysis that presents a very good alternative to the correlated approximation limited only to low frequencies.

3.2 Comparison and validation with BEM/FEM results

The method was validated by comparison with results obtained from a detailed solution using commercial FEM/BEM software to solve the vibro-acoustic response to diffuse sound loading. Comparison with test data is also provided as illustrated on Figure 9. The structure is a lightweight payload component subjected to fairing acoustic loads. FE model, BEM/FEM and test results were provided by industrial partner. Acceleration PSD response is considered at measurement location and plotted over the [0-2000]Hz frequency band.

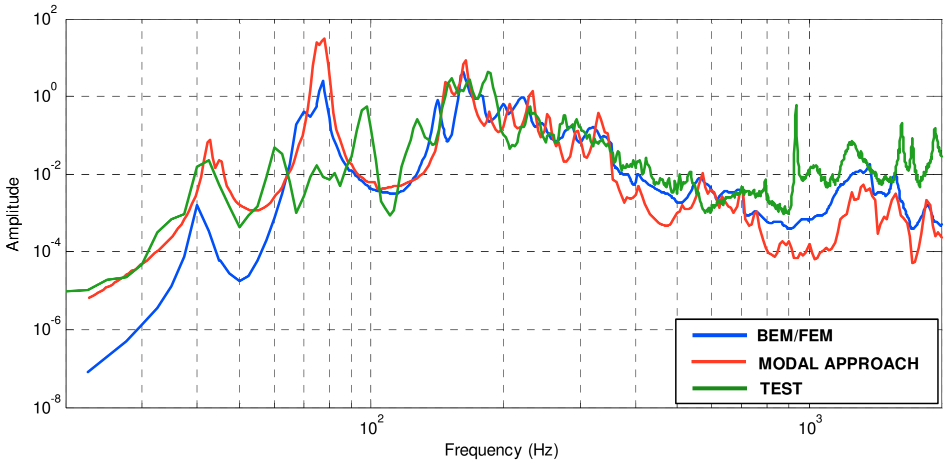


Figure 9 : acceleration PSD comparison with BEM/FEM software and test results

Figure 9 shows that similar results are obtained with the detailed BEM/FEM solution and the modal approach described in this paper. Correlation is very good in the mid frequency range. Good correlation with test data is also illustrated at low frequencies, for the first structural mode. Poorer correlation is found around 70Hz and at higher frequencies. However it should be noted that BEM/FEM results also do not perfectly fit test results. Adjustment of modal parameters to test data could also significantly improve the accuracy of the modal approach that already provides very satisfying results.

3.3 Industrial application: IXV Thermal Protection Panels (TPS)

An industrial application of the method is presented for ESA re-entry vehicle IXV (Intermediate eXperimental Vehicle). Snecma Propulsion Solide is in charge of the design and manufacturing of the vehicle's ablative Thermal Protection System (TPS) for nose and windward assemblies. The most severe load case for the TPS is the re-entry thermo-mechanical environment, with very high levels of heat-flux. However it is also critical for the mission that structural integrity is preserved during lift-off with regards to the launcher's acoustic environment. The IXV Nose and Windward TPS Assembly are illustrated on Figure 10 and Figure 11.

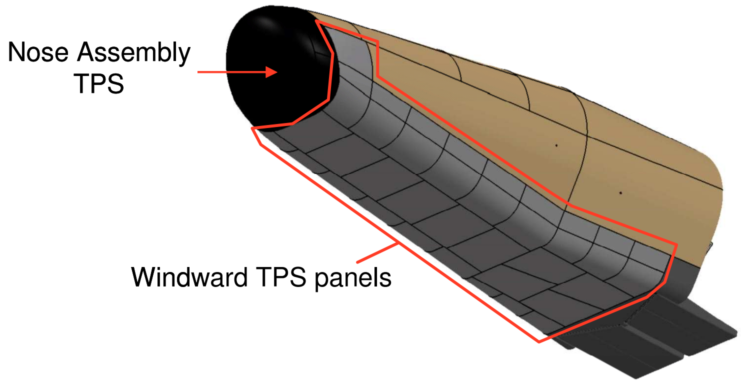


Figure 10 : ESA IXV spacecraft and Thermal Protection Systems for atmospheric re-entry

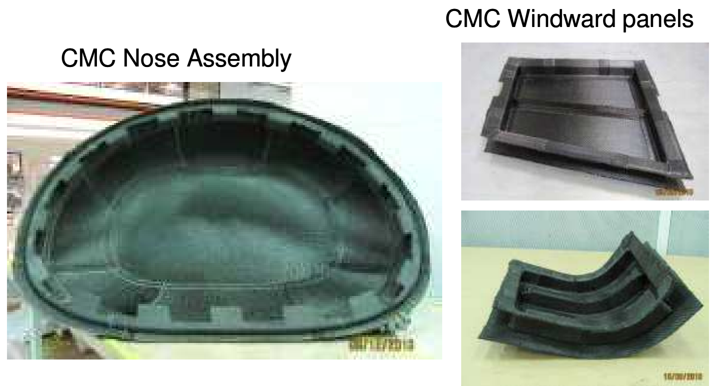


Figure 11 : CMC Nose cap and panels manufactured by Snecma Propulsion Solide

CMC panels are shown on Figure 11 without the internal insulation. Composite TPS panels are lightweight structures with a large surface that will be exposed to diffuse acoustic noise inside the fairing during lift-off and atmospheric flight. The IXV reference mission plans a launch on Arianespace's VEGA launcher. Acoustic environment inside the fairing is specified in the VEGA user manual [5] and is listed in Table 2 below.

Table 2: VEGA reference acoustic noise spectrum under the fairing

Octave center frequency (Hz)	Flight limit SPL (dB) Pref = 2.10^{-5} Pa
31.5	124
63	129
125	135
250	132
500	131
1000	120
2000	100
OASPL (20-2828Hz)	138.5

Diffuse acoustic loading is derived from VEGA requirements and the methodology described above is used to obtain RMS stresses in the CMC panels and RMS forces in stand-off attachments to the rest of the structure (spacecraft cold structure). Design justification is performed by considering CMC material strength properties and maximum allowable stand-off forces. Illustration is provided for the nose assembly panel that presents a large CMC surface exposed to the acoustic field. On Figure 12, FE mesh with shell elements is presented for the CMC part of the nose assembly. Orthotropic material properties are assigned to CMC shell elements.

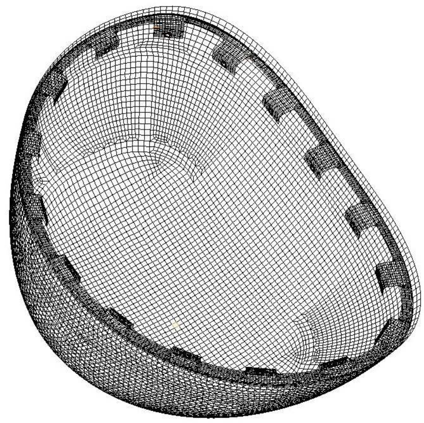


Figure 12 : FE mesh for IXV nose assembly CMC TPS

Internal insulation is not included on Figure 12 but is represented in FE models by 3D solid elements. Total size for the Nose FE model is about 700 000 dof with around 15 000 wetted nodes on the exposed surface. Joint-acceptance terms are calculated after a normal mode analysis, with 79 modes selected in the [0-2000]Hz frequency band using the mode filtering feature. The feature was very useful to remove many of the insulation local modes than do not participate in the joint-acceptance matrix. RMS stress contour plots (normalized to 100) are generated within Patran MSC.Random module and are illustrated on Figure 13.

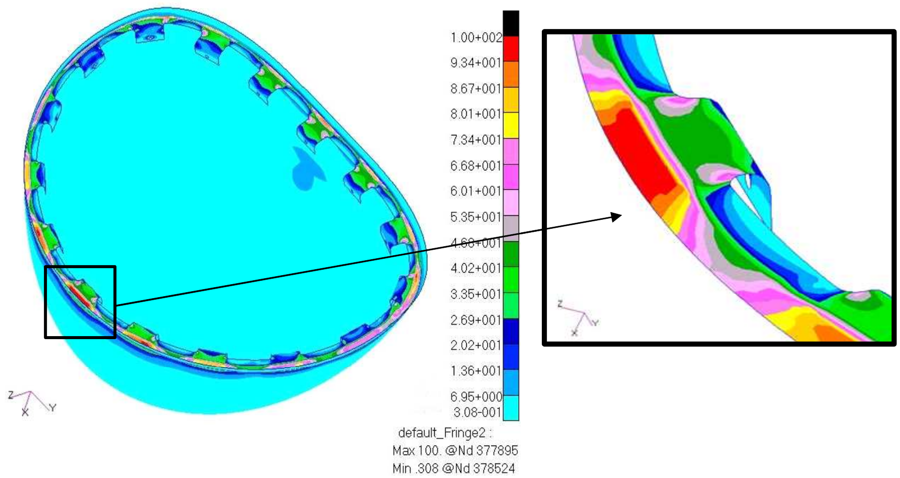


Figure 13 : RMS S_{11} stress contour plot for the CMC nose assembly under acoustic loading

RMS stress contour plots are used to determine stress concentration areas on the CMC parts. Maximum stresses are included in a design justification loop in order to assess TPS structural integrity under acoustic loading with appropriate safety margins.

4. Conclusions

The modal condensation approach presented in this paper to compute the structural response of structures subjected to diffuse acoustic loading appeared to be a very attractive formulation when radiated sound pressure can be neglected. It is especially well suited for quick but accurate predictions during preliminary design phases. Useful innovative features such as mode filtering or possibility to omit cross joint-acceptance terms have been included and allow computations with limited resources. More detailed solution is also possible, such as condensation with normal modes computed with the inclusion of virtual mass air properties on the wetted element. The methodology has been presented for the diffuse sound field correlation, but it can be very easily extended to other correlation functions (e.g.TBL acoustic noise, propagating waves field) or test-generated data.

Development of the pre-processor has been made in order to enable the use of standard FE solver MSC.Nastran and MSC.Patran module MSC.Random. This provides the user with access to great post-processing capabilities such as RMS contour plots, very useful in the design process for identification of critical zones. It should also be noted that the tool could be integrated in a FE-based fatigue analysis with PSD input.
An industrial application has been illustrated for the ESA IXV thermal protection systems (nose and windward) subjected to a diffuse acoustic environment at launcher lift-off. This application demonstrated that the tool was able to handle large FE model with more than 700 000 dof and can be effectively integrated in the design justification procedure. Future work includes the improvement of the formulation for un-baffled structures with acoustic pressure acting on both sides of the exposed surface.

5. References

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