DYNAMIC ANALYSIS OF FE MODELS WITH FLUID CAVITIES FOR IMPROVED CORRELATION WITH VIBRATION TESTS

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ABSTRACT

Fluid cavities are present in various structures such as pressurized tanks, ship ballasts and vehicle compartments. Although fluid cavities can be easily modelled using existing finite element codes, subsequent dynamic analysis of the coupled system can be time consuming and less accurate due to the properties of the coupled system and in particular the loss of symmetry and the presence of complex modes.

A real mode approach has been formulated and implemented to perform coupled analysis of structures with fluid cavities using the standard capabilities of finite element codes. The real mode approach allows the straightforward use of modal energies, modal effective parameters and frequency response functions (FRF). In addition, the method has been extended to include new features such as sensitivity analysis and the elaboration of condensed models with fluid cavities.

1. INTRODUCTION

Frequency responses calculations of a structure coupled with acoustic cavities are in general computationally intensive due to the unsymmetrical form of the coupled equations of motion which are often solved directly at each frequency step. To reduce the computational effort, the system can be reduced in size via a modal transformation using the *uncoupled* normal modes of the structure and acoustic cavities. Unfortunately, the resulting condensed system may suffer in accuracy due to modal truncation errors - especially when dealing with heavy fluids such as water or highly pressurized gases. To overcome this problem, residual modes [1] can be added to the normal modes of both the structure and fluid cavities to compensate for truncation effects.

A real mode approach using residual modes was initially introduced some years ago [2-3] has been implemented in the PRIMODAL analysis tool [4].

Recently, a vibration test campaign was carried out on a launcher sub-component mounted on a shaker table. Comparison of the measurements with the FE model of the structure without fluid showed the necessity to account for the fluid cavity within the structure. The use of the above method applied to a FE model including the fluid cavity greatly improved the correlation with the vibration tests.

In the first part of this paper the theoretical formulation of the method is presented followed by a discussion of modal energies and the elaboration of condensed models in the presence of fluid coupling (Top Modal). Finally, the example involving the industrial structure vibration test and model correlation is presented (ArianeGroup).

2. THEORY

2.1. Coupled Equations of Motion

The equations of motion governing the harmonic response of a structure comprising s degrees of freedom (DOF) coupled with one or more fluid cavities comprising f DOF are expressed below.

$$\left(-\omega^2 \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0}_{sf} \\ -\mathbf{A}_{fs} & \mathbf{M}_{ff} \end{bmatrix} + i\,\omega \begin{bmatrix} \mathbf{C}_{ss} & \mathbf{0}_{sf} \\ \mathbf{0}_{fs} & \mathbf{C}_{ff} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{A}_{sf} \\ \mathbf{0}_{fs} & \mathbf{K}_{ff} \end{bmatrix} \right) \begin{bmatrix} \mathbf{u}_s \\ \mathbf{p}_f \end{bmatrix} = \begin{bmatrix} \mathbf{F}_s \\ \dot{\mathbf{Q}}_f \end{bmatrix} \quad (1)$$

with:

$\mathbf{M}_{ss}$	Structure mass matrix (sym.)
$\mathbf{C}_{ss}$	Structure viscous damping matrix (sym.)
$\mathbf{K}_{ss}$	Structure stiffness matrix (sym.)
$\mathbf{M}_{ff}$	Fluid "mass" matrix (sym.)
$\mathbf{C}_{ff}$	Fluid viscous damping matrix (sym.)
$\mathbf{K}_{ff}$	Fluid "stiffness" matrix (sym.)
$\mathbf{A}_{fs}$	Coupling matrix ($\mathbf{A}_{sf} = \mathbf{A}_{fs}^T$)
$\mathbf{u}_s$	Vector of structural displacements
$\mathbf{p}_f$	Vector of fluid pressures
$\mathbf{F}_s$	Vector of forces applied to the structure
$\mathbf{Q}_f$	Vector of acoustic sources ($\dot{\mathbf{Q}}_f = i\omega\mathbf{Q}_f$)

Damping may be introduced in the fluid cavities using acoustic absorbers (matrix $\mathbf{C}_{ff}$), or as hysteretic damping, η_f, which may be added to the stiffness matrix using $(1+i\eta_f)\mathbf{K}_{ff}$. Similarly, the structure may include both viscous and structural (hysteretic) damping.

Eq. (1) may be solved directly at each frequency, ω, however the computation time may be overly prohibitive for large models (10^5 or more DOF).

As an alternative, the modal reduction technique described hereafter has been developed to substantially reduce the computational effort.

2.2. Uncoupled Normal Modes

The first step of the reduction process consists of computing the uncoupled normal modes of the structure and acoustic cavities as defined by the eigenvalue problems shown below.

$$(-\omega_n^2 \mathbf{M}_{ss} + \mathbf{K}_{ss})\mathbf{\Phi}_{sn} = \mathbf{0}_s \qquad (2a)$$

$$(-\omega_m^2 \mathbf{M}_{ff} + \mathbf{K}_{ff})\mathbf{\Phi}_{fm} = \mathbf{0}_f \qquad (2b)$$

This computation is relatively fast since each system is real and symmetric and may therefore be solved efficiently using for example the Lanczos method. The number of modes (*n* structural modes and *m* fluid modes) must include all modes up to the highest excitation frequency.

2.3. Residual Modes

To minimize truncation errors, the structure and fluid normal modes $\mathbf{\Phi}_{sn}$ and $\mathbf{\Phi}_{fm}$ must be enriched by a set of residual modes that provide information about the coupling effects across the structure-fluid cavity boundaries.

A residual mode is similar to a normal mode in that it satisfies the same orthogonality properties and has an associated eigenvalue. However, it does not satisfy the eigenvalue problem since each residual is in fact a particular linear combination of *all* the truncated (superior) normal modes.

Although residual modes (sometimes known as residual vectors or pseudo-modes) have been in use for well over two decades, their application to coupled analysis is recent [1] and has been implemented in this study. The procedure for deriving the residual modes is as follows.

For the structure, a set of *m* static modes $\mathbf{X}_{sm}$ is computed resulting from the forces exerted by the fluid modes across the fluid-structure boundary.

$$\mathbf{K}_{ss}\,\mathbf{X}_{sm} = \mathbf{C}_{sf}\,\mathbf{\Phi}_{sm} \qquad (3)$$

Similarly, for the fluid, a set of *n* static modes $\mathbf{X}_{fn}$ is obtained using the pressure exerted by the structure modes across the same boundary but in the opposite sense.

$$\mathbf{K}_{ff}\,\mathbf{X}_{fn} = \mathbf{C}_{fs}\mathbf{\Phi}_{fn} \qquad (4)$$

Next, the static modes $\mathbf{X}_{sm}$ and $\mathbf{X}_{fn}$ are "filtered" or rendered orthogonal with respect to the normal modes $\mathbf{\Phi}_{sn}$ and $\mathbf{\Phi}_{fm}$, and then orthogonalized to form an orthonormal basis of residual modes $\hat{\mathbf{\Phi}}_{sm}$ and $\hat{\mathbf{\Phi}}_{fn}$ which are then appended to the normal modes to form enriched modal bases. Note that for simplicity we use the same notation $\mathbf{\Phi}_{sn}$ and $\mathbf{\Phi}_{fm}$ to designate the enriched set of modes.

$$\mathbf{\Phi}_{sn} \leftarrow [\mathbf{\Phi}_{sn} \quad \hat{\mathbf{\Phi}}_{sm}] \qquad (5a)$$

$$\mathbf{\Phi}_{fm} \leftarrow [\mathbf{\Phi}_{fm} \quad \hat{\mathbf{\Phi}}_{fn}] \qquad (5b)$$

2.4. Coupled Normal Modes

The normal modes of the coupled system of Eq. (1) are obtained by applying the following transformation involving the uncoupled modes and associated residual modes of Eq. (5).

$$\mathbf{u}_s = \mathbf{\Phi}_{sn}\,\mathbf{q}_n \quad \text{and} \quad \mathbf{u}_f = \mathbf{\Phi}_{fm}\,\mathbf{q}_m \qquad (6)$$

This leads to the following pair of unsymmetric eigenvalues problems combining the structure and fluid modes *b* = *n* + *m*.

$$(-\omega_k^2 \mathbf{M}_{bb} + \mathbf{K}_{bb})\mathbf{\Phi}_{bk} = \mathbf{0}_{bk} \qquad (7a)$$

$$\mathbf{Y}_{kb}(-\omega_k^2 \mathbf{M}_{bb} + \mathbf{K}_{bb}) = \mathbf{0}_{kb} \qquad (7b)$$

with

$$\mathbf{M}_{bb} = \begin{bmatrix} \mathbf{\Phi}_{ns}\,\mathbf{M}_{ss}\,\mathbf{\Phi}_{sn} & \mathbf{0}_{nm} \\ -\mathbf{\Phi}_{mf}\,\mathbf{A}_{fs}\mathbf{\Phi}_{sn} & \mathbf{\Phi}_{mf}\,\mathbf{M}_{ff}\,\mathbf{\Phi}_{fm} \end{bmatrix} \qquad (8)$$

$$\mathbf{K}_{bb} = \begin{bmatrix} \mathbf{\Phi}_{ns}\,\mathbf{K}_{ss}\,\mathbf{\Phi}_{sn} & \mathbf{\Phi}_{ns}\mathbf{A}_{sf}\mathbf{\Phi}_{fm} \\ \mathbf{0}_{mn} & \mathbf{\Phi}_{mf}\,\mathbf{K}_{ff}\,\mathbf{\Phi}_{fm} \end{bmatrix} \qquad (9)$$

The right eigenvectors $\mathbf{\Phi}_{bk}$ are obtained from Eq. (7a) whereas the left eigenvectors $\mathbf{Y}_{kb}$ are obtained from Eq. (7b). These eigenvectors can be expressed in terms of the physical DOF *s* and *f* using the transformation of Eq. (6) as shown below.

$$\begin{bmatrix} \mathbf{\Phi}_{sk} \\ \mathbf{\Phi}_{fk} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{sn} \\ \mathbf{\Phi}_{fm} \end{bmatrix} \mathbf{\Phi}_{bk} \qquad \begin{bmatrix} \mathbf{Y}_{sk} \\ \mathbf{Y}_{fk} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{sn} \\ \mathbf{\Phi}_{fm} \end{bmatrix} \mathbf{Y}_{bk} \qquad (10)$$

Due to the particular form of the system's asymmetry, it can be shown that the components of the right and left eigenvectors respect the following relation.

$$\begin{bmatrix} \mathbf{Y}_{sk} \\ \mathbf{Y}_{fk} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{sk} \\ \mathbf{\Phi}_{fk} / \omega_k^2 \end{bmatrix} \quad (11)$$

The generalized masses are obtained from the orthogonality relation involving both right and left eigenvectors.

$$m_k = \mathbf{Y}_{kb} \mathbf{M}_{bb} \mathbf{\Phi}_{bk} \quad (12a)$$
$$k_k = \mathbf{Y}_{kb} \mathbf{K}_{bb} \mathbf{\Phi}_{bk} \quad (12b)$$

2.5. Modal Energies and Modal Damping

The modal energies are obtained from the generalized masses and stiffnesses in Eq. (12) and using the relationship between the left and right eigenvectors of Eq. (11). This leads to the following expressions for the kinetic and elastic energies.

$$T_k = \frac{1}{2} m_k \omega_k^2 = \frac{1}{2}\left(\mathbf{Y}_{ks}\mathbf{M}_{ss}\mathbf{\Phi}_{sk} + \mathbf{Y}_{kf}\mathbf{M}_{ff}\mathbf{\Phi}_{fk} - \mathbf{\Phi}_{kf}\mathbf{A}_{fs}\mathbf{\Phi}_{sk} \right) \quad (13a)$$

$$U_k = \frac{1}{2} k_k = \frac{1}{2}\left(\mathbf{Y}_{ks}\mathbf{K}_{ss}\mathbf{\Phi}_{sk} + \mathbf{Y}_{kf}\mathbf{K}_{ff}\mathbf{\Phi}_{fk} + \mathbf{\Phi}_{ks}\mathbf{A}_{sf}\mathbf{\Phi}_{fk} \right) \quad (13b)$$

From Eq. (13) we see that the modal energies are expressed as a sum of energies from the structure and fluid parts of the system along with a coupling term which is the same for the kinetic and elastic energies but opposite in sign. The fraction of energy in the coupled term can be used as a relative measure of fluid/structure coupling for a given mode.

Using the elastic energy, the following expression for the equivalent modal damping g_k can be derived based on the physical damping in the structure g_s and in the fluid g_f.

$$g_k = \frac{g_s\left(\mathbf{Y}_{ks}\mathbf{K}_{ss}\mathbf{\Phi}_{sk}\right) + g_f\left(\mathbf{Y}_{kf}\mathbf{K}_{ff}\mathbf{\Phi}_{fk}\right)}{m_k \omega_k^2} \quad (14)$$

2.6. Static and Modal Parameters

Let us consider the coupled system with given boundary conditions defined by junction DOF j and internal DOF i. We shall use the subscript a to represent all DOF ($a = i + j$).

The right and left eigenvectors of Eq. (7) are related to the asymmetric mass and stiffness matrices $\mathbf{M}_{ii}$ and $\mathbf{K}_{ii}$ as follows

$$(-\omega_k^2 \mathbf{M}_{ii} + \mathbf{K}_{ii})\mathbf{\Phi}_{ik} = \mathbf{0}_{ik} \quad (15a)$$

$$\mathbf{Y}_{ki}(-\omega_k^2 \mathbf{M}_{ii} + \mathbf{K}_{ii}) = \mathbf{0}_{ki} \quad (15b)$$

and have the following generalized mass and stiffness.

$$m_k = \mathbf{Y}_{ki}\mathbf{M}_{ii}\mathbf{\Phi}_{ik} \quad k_k = \mathbf{Y}_{ki}\mathbf{K}_{ii}\mathbf{\Phi}_{ik} \quad k_k = m_k \omega_k^2 \quad (16)$$

The static junction modes, obtained by imposing a unit displacement successively at each junction DOF j are given by:

$$\mathbf{\Psi}_{jj} = \mathbf{I}_{jj} \quad \mathbf{\Psi}_{ij} = -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ij} \quad \mathbf{\Psi}_{ji} = -\mathbf{K}_{ji}\mathbf{K}_{ii}^{-1} \quad (17)$$

The mass and stiffness matrices condensed at the junction are given by:

$$\overline{\mathbf{M}}_{jj} = \mathbf{\Psi}_{ja}\mathbf{M}_{aa}\mathbf{\Psi}_{aj} \quad \overline{\mathbf{K}}_{jj} = \mathbf{\Psi}_{ja}\mathbf{K}_{aa}\mathbf{\Psi}_{aj} \quad (18)$$

And finally, the modal participation factors are given by:

$$\mathbf{L}_{kj} = \mathbf{Y}_{ka}\mathbf{M}_{aa}\mathbf{\Psi}_{aj} \quad \mathbf{L}_{jk} = \mathbf{\Psi}_{ja}\mathbf{M}_{aa}\mathbf{\Phi}_{ak} \quad (19)$$

2.7. Frequency Response Functions

The frequency response functions (FRF) of the coupled system are expressed using mode superposition theory. The contribution of each mode is defined in terms of its natural frequency, damping and modal effective parameters.

Details concerning the use of modal effective parameters in mode superposition can be found in [5].

The same formulas apply here with the exception that transposed vectors are replaced by the left-side vectors. The FRF for flexibility, transmissibility (displacement and force) and mass are defined below along with the corresponding modal effective parameters.

$$\mathbf{G}_{ii}(\omega) = \sum_k H_k(\omega)\tilde{\mathbf{G}}_{ii,k} \qquad \tilde{\mathbf{G}}_{ii,k} = \frac{\mathbf{\Phi}_{ik}\mathbf{Y}_{ki}}{m_k \omega_k^2} \quad (20)$$

$$\mathbf{T}_{ij}(\omega) = \sum_k T_k(\omega)\tilde{\mathbf{T}}_{ij,k} \qquad \tilde{\mathbf{T}}_{ij,k} = \frac{\mathbf{\Phi}_{ik}\mathbf{L}_{kj}}{m_k} \quad (21)$$

$$\mathbf{T}_{ji}(\omega) = \sum_k T_k(\omega)\tilde{\mathbf{T}}_{ji,k} \qquad \tilde{\mathbf{T}}_{ji,k} = \frac{\mathbf{L}_{jk}\mathbf{Y}_{ki}}{m_k} \quad (22)$$

$$\mathbf{M}_{jj}(\omega) = \sum_k T_k(\omega)\tilde{\mathbf{M}}_{jj,k} \qquad \tilde{\mathbf{M}}_{jj,k} = \frac{\mathbf{L}_{jk}\mathbf{L}_{kj}}{m_k} \quad (23)$$

The functions $H_k(\omega)$ and $T_k(\omega)$ are the usual dimensionless amplification factors expressed as a function of the natural frequency ω_k and damping ζ_k of each mode k.

$$H_k(\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_k}\right)^2 + i\,2\zeta_k \frac{\omega}{\omega_k}} \qquad (24)$$

$$T_k(\omega) = \frac{1 + i\,2\zeta_k \frac{\omega}{\omega_k}}{1 - \left(\frac{\omega}{\omega_k}\right)^2 + i\,2\zeta_k \frac{\omega}{\omega_k}} \qquad (25)$$

The above expressions for FRF assume uncoupled modal damping. Also, note that because of certain symmetries in the equations, the FRF involving only structure or only fluid DOF of any type ($\mathbf{G}$, $\mathbf{T}$ or $\mathbf{M}$) are symmetric.

Similarly, "mixed" FRF involving a fluid and a structure DOF are related as follows for $\mathbf{X}$ of any type ($\mathbf{G}$, $\mathbf{T}$ or $\mathbf{M}$).

$$\mathbf{X}_{fs}(\omega) = \omega^2 \mathbf{X}_{sf}(\omega) \qquad (26)$$

2.8. Condensed Models

Condensed models based on the Craig-Bampton formulation can be elaborated directly from the preceding static and modal terms. The Craig-Bampton mass and stiffness matrices are shown below.

$$\mathbf{M}_{cc} = \begin{bmatrix} \mathbf{m}_k & \mathbf{L}_{kj} \\ \mathbf{L}_{jk} & \overline{\mathbf{M}}_{jj} \end{bmatrix} \qquad \mathbf{K}_{cc} = \begin{bmatrix} \mathbf{k}_k & \mathbf{0}_{kj} \\ \mathbf{0}_{jk} & \overline{\mathbf{K}}_{jj} \end{bmatrix} \qquad (27)$$

Even though the above condensed matrices are in general asymmetric they remain compatible with most FE analysis codes. For example, using NASTRAN asymmetric matrices can be written using the M2PP and K2PP entries.

3. INDUSTRIAL APPLICATION

3.1. Introduction

The development of a solid rocket motor (SRM) requires both identification testing and dynamic qualification tests. Identification tests are performed to determine the dynamic behavior of the motors and to improve and validate the corresponding FE models. The goal of qualification tests is to apply the same vibrational environments (sine, random, shock) that the motor will be subjected to during the different phases of its life cycle and to verify that it can withstand the loads.

The test campaigns are performed using shakers along with a large number of sensors representing over 100 measurement channels for acceleration, strain, temperature, pressure, displacement, etc. A typical SRM in test configuration is shown below along side the associated FE model.

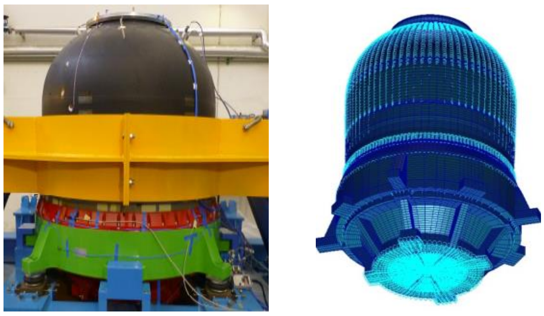


Figure 1: SRM in Test Configuration with FE model

3.2. Mechanical Behavior of Propellant

The main component of these SRM is the propellant whose mechanical behavior is particular due to the viscoelastic and incompressible nature of the material. The propellant plays a significant role in the first global modes of the structure with large effective mass such as the first axial and bending modes. To evaluate the viscoelastic properties of the propellant, temperature conditioning is performed.

The comparison of measured FRF (located at the same position) at two different temperatures showed that the dynamic behavior of the motor appears to change in nature. Changes in the response peaks were not limited to simple shifts in frequency (higher or lower) as a function of the temperature. As an illustration, two frequency responses are shown in Figure 2 at two locations in the motor (front and mid-height) for two different test temperatures.

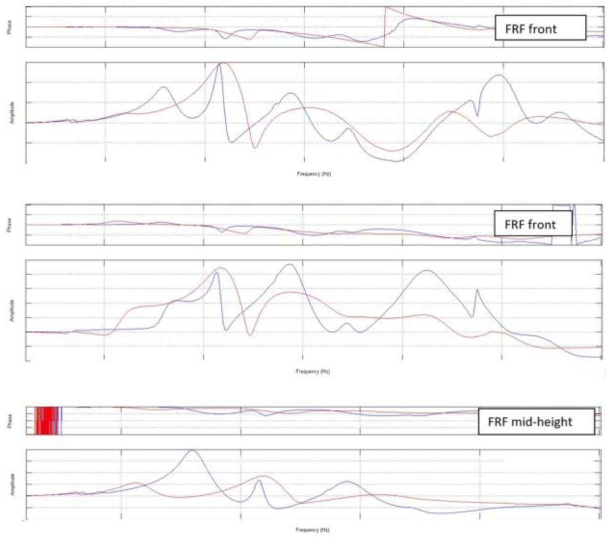


Figure 2: FRF at Two Different Temperatures

3.3. Influence of Air Cavities

Test performed with and without internal pressure confirmed the assumption that the internal air cavity had a strong influence on the dynamic responses of the motor. This internal cavity changes as a function of the temperature. The propellant retracts and contracts while maintaining a constant volume. This in turn significantly changes the internal air cavities and as a consequence the coupled fluid-structure behavior.

As an illustration, the change in the FRF as a function of the internal pressure is displayed in Figure 3. The green curve corresponds to no internal pressure whereas the blue and red curves correspond to two different values of pressure. Note that with no pressure, two modes are present between 0 and 150 Hz whereas a 3rd mode appears when a pressure is applied.

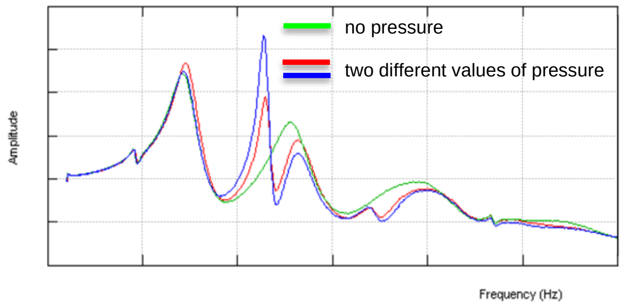


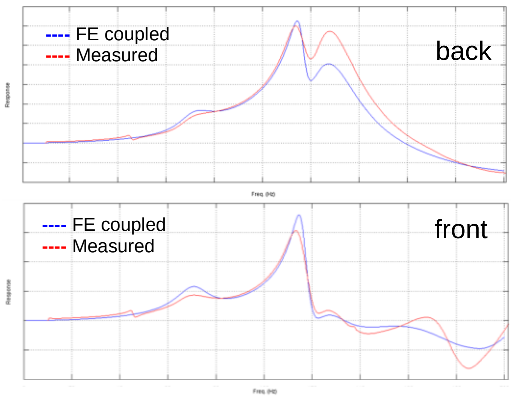
Figure 3: Appearance of Coupled Modes as a Function of Internal Pressure

Up until now, the air cavities were not modeled in the FE models. These experimental observations indicated the importance of representing the cavities in the model in order to correctly simulate the measured frequency responses.

3.4. Coupled Analysis Results

The coupled fluid-structure approach was developed by Top Modal and implemented in PRIMODAL [4] as part of a development for ArianeGroup. The use of this coupled FE approach with explicit modeling of the air cavity in the motor resulted in greatly improved test/analysis correlation in terms of the FRF and a better understanding of the dynamic behavior of the SRM.

To illustrate this point, the FRF obtained from FE coupled analysis (blue curve) are compared with the measured responses (red curve) in Figure 4 at three different locations (front, back and mid-height). The correlation is good over the entire frequency range.



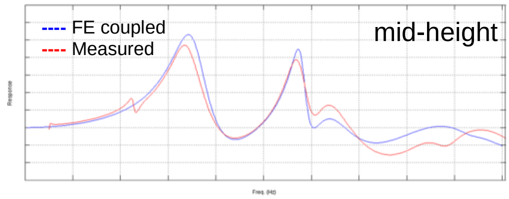


Figure 4: Comparison of FE and Measured FRF

4. CONCLUSIONS

A real mode approach to perform coupled analysis of structures with fluid cavities is shown to be numerically efficient and accurate thanks to the use of residual modes. Moreover, the approach is compatible with the standard capabilities of most finite element codes. The real mode approach allows the straightforward use of modal energies, modal effective parameters and frequency response functions (FRF).

Taking into account the air cavities via coupled analysis and the use of analysis tools in PRIMODAL (sensitivities, reduced models, responses) has greatly improved the quality of the FE models and the understanding of the dynamic behavior of the SRM.

The use of condensed models compatible with fluid cavities is also useful in the context of coupled analysis with the launcher.

5. REFERENCES

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