DYNAMIC ANALYSIS OF PARAMETERIZED MODELS USING RESIDUAL MODES

Nicolas ROY (1), Tiana BRAULT (2), Thomas LARROQUE (3)

(1) Top Modal, 130 rue Galilée, 31670 Labège, France, Email: nicolas.roy@topmodal.fr
(2) ArianeGroup, Les Cinq Chemins, 33185 Le Haillan, France, Email: tiana.brault@ariane.group
(3) Ingeliance Technologies, 6 rue Nicolas Leblanc, 33700 Mérignac, France, Email: thomas.larroque@ingeliance.com

KEYWORDS

residual modes, reduced models, mode superposition, modal synthesis, damping, viscoelastic material

ABSTRACT

Mode superposition theory is widely used in both analytic and experimental structural dynamics and provides a simple and effective way of representing the dynamic behavior of a structure. In general, a truncated set of normal modes is used along with a set of residual modes (sometimes referred to as residual vectors) which approximate the contribution of the neglected (truncated) higher frequency modes.

The role of the residual modes becomes even more important when performing perturbation analysis and structural modification or when taking into account frequency-dependent materials such as elastomers.

In some cases, it is desirable to consider large perturbations applied to entire sections or components of the structure and corresponding to different configurations of the structure such as a type of honeycomb panel or a choice of material. The standard approach is to elaborate a separate FE model for each structural configuration.

Recently, a new approach based on residual modes has been developed which allows combining several configurations into a single model by transforming the normal modes of each configuration into equivalent residual modes. In this manner a single parameterized model may be used to analyze the structure for any given configuration. The method can also be extended to the generation of parameterized condensed models.

1. INTRODUCTION

The Mode Acceleration Method, [1] has been widely used for over several decades to compensate for modal truncation errors in dynamic analysis. In this approach, static residual terms are used representing the quasi-static contribution of the non-retained modes whose natural frequencies are assumed greater than the excitation frequencies considered.

Starting in the early 1990's an alternative approach referred to as the Modal Truncation Augmentation Method, [2, 3] was proposed. This method appends additional modes, often referred to as residual modes or residual vectors, to the retained modes to form an enriched Ritz basis. This approach presents several advantages over Mode Acceleration including 1) ease of implementation since the residual modes are treated the same way as the normal modes, and 2) a more accurate dynamic representation of the contribution of the non-retained modes.

Detailed information concerning the Modal Truncation Augmentation Method including comparisons with the Mode Acceleration Method may be found in several references such as [4].

The use of residual modes has become common practice in various structural analysis techniques involving response calculation, modal identification, damping and fluid-structure coupling. Examples of these applications including background theory can be found in [5].

This paper focuses on the use of residual modes in the context of parameterized models. The idea is to elaborate a single model capable of accounting for a range of stiffness and damping values associated with a given structural part or material.

The paper provides a brief revue of residual modes before presenting the theory behind the parameterized models (Top Modal / Ingeliance). An industrial application of a launcher sub-component with parameterized stiffnesses in the solid propellant is used to illustrate the interest of the method (ArianeGroup).

2. THEORY

2.1. Frequency Responses

Consider a discretized model with internal and junction degrees of freedom i and j represented by a set of fixed-junction normal modes $\mathbf{\Phi}$ and static junction modes $\mathbf{\Psi}$. The transformation between physical and generalized displacements $\mathbf{u}_i$ and $\mathbf{q}_k$ is shown below in Eq. (1).

$$\begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{ik} & \mathbf{\Psi}_{ij} \\ \mathbf{0}_{jk} & \mathbf{I}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{q}_k \\ \mathbf{u}_j \end{bmatrix} \tag{1}$$

Applying the transformation (1) to the physical equations of motion leads to the following condensed equations of motion expressed in terms of the junction and generalized displacements $\mathbf{u}_j$ and $\mathbf{q}_k$.

$$\left(-\omega^2 \begin{bmatrix} \mathbf{m}_k & \mathbf{L}_{kj} \\ \mathbf{L}_{jk} & \overline{\mathbf{M}}_{jj} \end{bmatrix} + \begin{bmatrix} \mathbf{k}_k & \mathbf{0}_{kj} \\ \mathbf{0}_{jk} & \overline{\mathbf{K}}_{jj} \end{bmatrix} \right) \begin{bmatrix} \mathbf{q}_k \\ \mathbf{u}_j \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{ki}\, \mathbf{F}_i \\ \mathbf{\Psi}_{ji}\, \mathbf{F}_i + \mathbf{F}_j \end{bmatrix} \tag{2}$$

$\mathbf{m}_k$	diagonal generalized mass matrix
$\mathbf{L}_{kj}$	matrix of participation factors
$\overline{\mathbf{M}}_{jj}$	condensed mass matrix
$\mathbf{k}_k$	diagonal generalized stiffness matrix
$\overline{\mathbf{K}}_{jj}$	condensed stiffness matrix

Damping can be added to Eq. (2) using for example modal damping or structural damping.

2.2. Parameterized Zones

Let us assume that the above model has one or several zones, z, whose stiffness and damping properties are to be parameterized as a function of the excitation frequency.

The parameterized stiffness and damping of each zone are defined using the Young's modulus $E_i(z)$ and structural damping $g_i(z)$ of zone z at the excitation frequency ω_i.

In addition, we shall define $E_{ref}(z)$ as the reference value of the Young's Modulus used in computing the modes $\mathbf{\Phi}$ and $\mathbf{\Psi}$ of the model. The reference value is typically chosen at a frequency near the middle of the range of excitation frequencies.

The introduction of the parameterized zones produces the following perturbation term which is to be added to the left-hand side of the equations of motion defined in Eq. (2).

$$\sum_z \alpha_i(z) \begin{bmatrix} \mathbf{k}_{kk}(z) & \mathbf{k}_{kj}(z) \\ \mathbf{k}_{jk}(z) & \mathbf{k}_{jj}(z) \end{bmatrix} \tag{3}$$

The matrix of Eq. (3) is obtained by projection of the physical stiffness matrix of each onto the normal and static modes of the model as shown in Eq. (4).

$$\begin{bmatrix} \mathbf{k}_{kk}(z) & \mathbf{k}_{kj}(z) \\ \mathbf{k}_{jk}(z) & \mathbf{k}_{jj}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{ki} & \mathbf{0}_{kj} \\ \mathbf{\Psi}_{ji} & \mathbf{I}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{ii}(z) & \mathbf{K}_{ij}(z) \\ \mathbf{K}_{ji}(z) & \mathbf{K}_{jj}(z) \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_{ik} & \mathbf{\Psi}_{ij} \\ \mathbf{0}_{jk} & \mathbf{I}_{jj} \end{bmatrix} \tag{4}$$

The unitless perturbation coefficient $\alpha_i(z)$ is defined below.

$$\alpha_i(v) = \frac{E_i(z)}{E_{ref}(z)} - 1 + i\; g_i(z) \times \frac{E_i(z)}{E_{ref}(z)} \tag{5}$$

Note that at the reference frequency, Eq. (5) reduces to the reference structural damping $\alpha_i(v) = i\; g_i(z)$.

The values $E_i(z)$ and $g_i(z)$ are generally interpolated from user-provided profiles (tables).

2.3. Residual Modes using Current Method

The projected perturbation as defined in Eq. (4) is prone to truncation error since the basis of normal modes cannot satisfactorily represent the stiffness and damping variations within the parameterized zones.

To reduce the truncation errors, it is necessary to compute a set of residual modes which are then appended to the normal modes as described hereafter.

First, the modal forces related to each zone are obtained by the following expression.

$$\mathbf{F}_{ik} = \left(\sum_z \mathbf{K}_{ii}(z) \right) \mathbf{\Phi}_{ik} \tag{6}$$

Next, the modal forces are used to compute a set of static displacements, $\mathbf{X}_{ik}$ defined in Eq. (7).

$$\mathbf{K}_{ii}\; \mathbf{X}_{ik} = \mathbf{F}_{ik} \tag{7}$$

The static vectors, $\mathbf{X}_{ik}$, are then transformed to residual modes, $\hat{\mathbf{\Phi}}_{ik}$, by rendering them orthogonal to the normal modes, $\mathbf{\Phi}_{ik}$, and to each other. The corresponding mathematical equations are omitted here for simplicity but can be found in [5].

Finally, an enriched modal basis is constructed by appending the residual modes to the initial normal modes as shown in Eq. (8).

$$\mathbf{\Phi}_{ik} \leftarrow \begin{bmatrix} \mathbf{\Phi}_{ik} & \hat{\mathbf{\Phi}}_{ik} \end{bmatrix} \tag{8}$$

The enriched modal basis of Eq. (8) can now be introduced in the projection matrix of Eq. (4).

2.4. Residual Modes using Multiple Mode Sets

In the preceding development, the residual modes are obtained by projecting the viscoelastic stiffness variations onto the basis of the normal modes as shown in Eq. (6). However, in the case of large stiffness variations in the zones, the modes of the perturbed model may be quite different from the initial modes, in which case the projection of Eq. (6) will produce truncation errors.

To overcome this problem, a new approach to obtaining residual modes is proposed. The idea is rather simple and consists of extracting the residual modes from the normal modes of the perturbed model.

To illustrate the method, consider a model with a single viscoelastic material whose Young's modulus $E(f)$ is plotted below in blue.

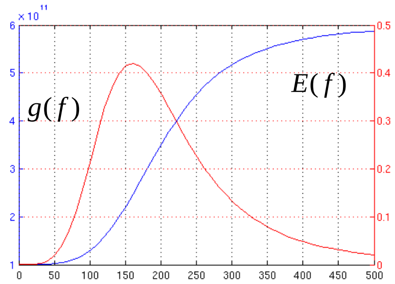


Figure 1: Viscoelastic zone properties

Using the current method described in §2.3, the modes of the model are computed using a reference value E_{ref} typically located within the frequency range of interest, often near the middle, in order to minimize the distance between the reference value and the min and max values of $E(f)$. The residual modes are then obtained using Eq. (6) to (8).

For small variations in $E(f)$, perhaps up to an order of magnitude, this approach works well. However, for larger variations that may span several orders of magnitude, another method is needed.

The new method consists of computing a *second* set of normal modes $\mathbf{\Phi}_{ik}^{+}$ at a modified value E_{ref}^{+} significantly different than E_{ref} such that the two values E_{ref} and E_{ref}^{+} span the range of $E(f)$. The normal modes $\mathbf{\Phi}_{ik}^{+}$ are then converted to residual modes $\hat{\mathbf{\Phi}}_{ik}^{+}$ and finally appended to the initial modes as depicted in Eq. (9).

$$\mathbf{\Phi}_{ik} \leftarrow \begin{bmatrix} \mathbf{\Phi}_{ik} & \hat{\mathbf{\Phi}}_{ik}^{+} \end{bmatrix} \qquad (9)$$

In the preceding description a single set of modes of the perturbed model with a single zone is considered. However, the new method can be extended to account for any number of zones and any number of perturbations thus resulting in multiple mode sets. These multiple mode sets can be treated together to form a single set of residual modes to be appended to the reference model.

3. IMPLEMENTATION

3.1. Introduction

The new method involving residual modes using multiple mode sets has been implemented using the NASTRAN finite element code along with the PRIMODAL structural analysis code. The procedure is performed in two steps – generation of mode sets followed by computation of residual modes. These two steps are schematized below in Figure 2 and described hereafter.

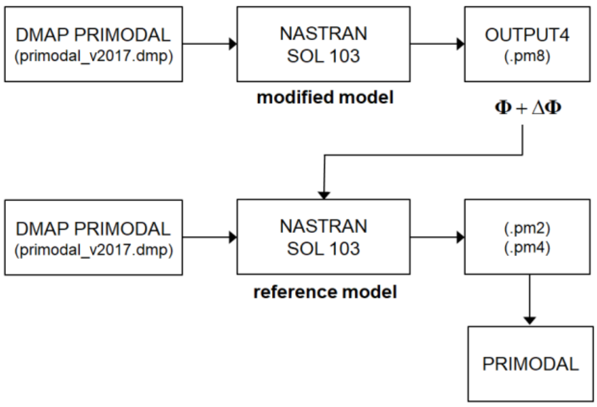


Figure 2: Two-Step Implementation

3.2. Generation of Mode Sets

The first step involves performing a modal analysis of the model for a given perturbation E_{ref}^{+} as schematized in the upper part of Figure 2. A DMAP script is used to output the perturbed normal modes $\mathbf{\Phi}_{ik}^{+}$ to an output file. This step is repeated for each desired perturbation.

3.3. Computation of Residual Modes

Once all of the mode sets have been generated in Step 1, a second modal analysis is performed using the reference model as depicted in the lower part of Figure 2.

The same DMAP script is used here to import all of the mode sets generated in Step 1, and convert them to residual modes according to the procedure presented in §2.4.

The DMAP script also exports the normal and residual modes as well as other modal terms (participation factors, condensed matrices, etc.) to a pair of output files.

3.4. Parameterized Frequency Responses

The NASTRAN output files are imported to PRIMODAL where the parameterized frequency responses are computed using the parameterized equations of motion described in §2.2.

Modal truncation errors remain small even over a large range of perturbations thanks to the presence of the multiple mode sets in the form of residual modes.

4. INDUSTRIAL APPLICATION

4.1. Model and Viscoelastic Material

A preliminary validation was performed in order to demonstrate the feasibility of the new method. The validation was performed on a solid rocket motor (SRM). The FE model of the SRM is shown in Figure 3. Two viscoelastic zones shown in Figure 4 were considered. Both zones are related to a single viscoelastic material.

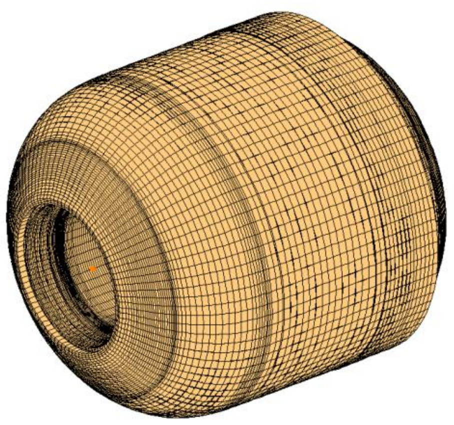


Figure 3: FE Model including Viscoelastic Zones

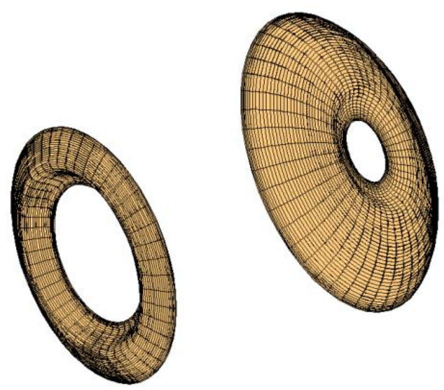


Figure 4: Viscoelastic Zones of FE Model

4.2. Parameterization

Two values were considered for the Young's modulus *E* of the viscoelastic material and are given in Figure 5. Note that the two values are different by several orders of magnitude. Two NASTRAN models, LOW and HIGH, were used corresponding to the two different values of the Young's modulus.

Model	*E*
LOW	1e2
HIGH	7.5e7

Figure 5: Values used for Young's Modulus

The influence of the Young's modulus on the dynamic mass in the X and Y directions is shown in Figure 6. The responses were computed using a global damping of g = 0.01 in order to better distinguish the peaks associated with the modes.

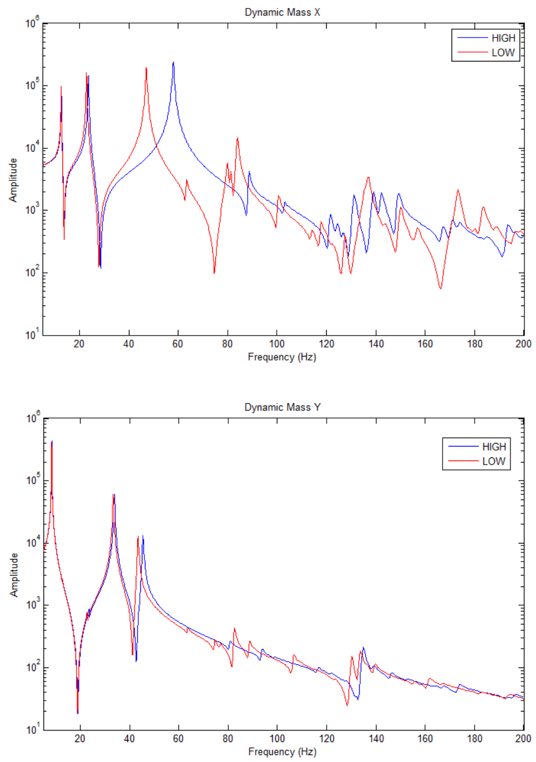


Figure 6: Influence of Young's Modulus on FRF

4.3. Validation of New Method

The validation consists of using the LOW model to estimate the frequency responses of the HIGH model via the new method based on the use of viscoelastic zones and multiple modal bases. The 2-step procedure is as follows:

Step 1: Modal analysis of the HIGH model and export of the modes $\mathbf{\Phi}_{ik}^{+}$.

Step 2: Modal analysis of the LOW model and computation of frequency responses using the residual modes obtained from $\mathbf{\Phi}_{ik}^{+}$ and a viscoelastic Young's modulus corresponding to the value of the HIGH model.

The FRF for the dynamic masses using the new method are plotted below in Figure 7 along with the FRF obtained directly from the HIGH model. The responses are nearly identical.

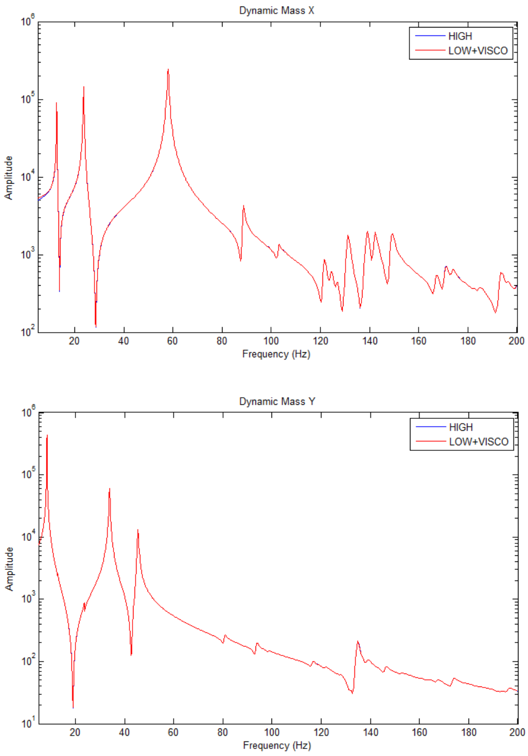
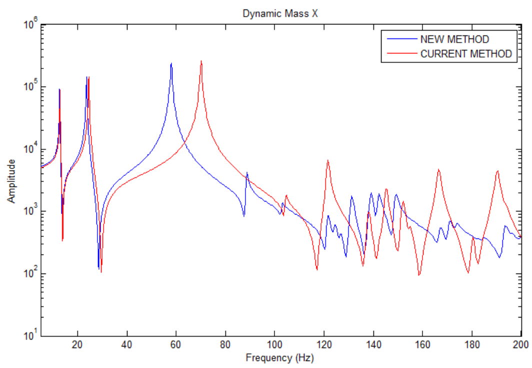


Figure 7: FRF Obtained using New Method

In order to evaluate the performance of the new method compared to the current method of §2.3, the FRF shown above were computed using the current method and compared with those obtained by the new method. The results are shown in Figure 8. The current method overshoots the frequency shifts in the low frequency range and shows large discrepancies in amplitude in the higher frequencies. These errors result from the large variation in the Young's modulus and the corresponding modes.



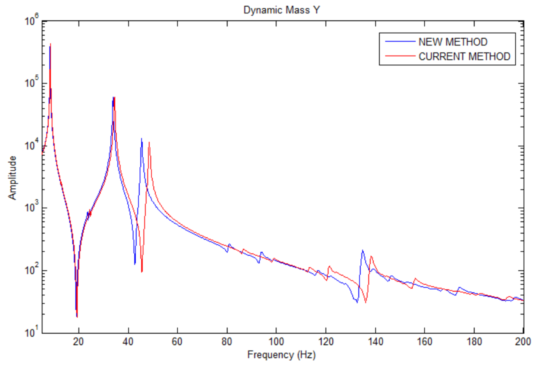


Figure 8: FRF – New vs Current Method

5. CONCLUSIONS

This preliminary study has illustrated the interest and feasibility of an alternative method for computing structural responses with frequency-dependent viscoelastic materials. Similar to the existing method, residual modes are used to account for the stiffness perturbations of the viscoelastic material. However, with the new method, the residual modes are obtained from a set of normal modes corresponding to a given perturbation, thus minimizing the influence of modal truncation errors.

The new method is especially effective when dealing with large perturbations of several orders of magnitude. Moreover, the method can manage multiple sets of modes to account for several viscoelastic materials or several perturbations for a given viscoelastic material

6. REFERENCES

1. Craig, R. R., *Structural Dynamics - An Introduction to Computer Methods*, John Wiley & Sons, 1981.

2. Rose, T., Using Residual Vectors in MSC/NASTRAN Dynamic Analysis to Improve Accuracy, MSC World Users' Conference, 1991.

3. Dickens, J. M. and Pool, K. V., Modal Truncation Vectors and Periodic Time Domain Analysis Applied to a Cyclic Symmetry Structure, *Comput. Struct*. 45(4), pp. 685-696, 1992.

4. Dickens, J. M., Nakagawa, J. M. and Wittbrodt, M. VJ., A Critique of Mode Acceleration and Modal Truncation Augmentation Methods for Modal Response Analysis, *Comput. Struct*. Vol. 62 No. 6, pp. 985-998, 1997.

5. Roy, N. and Girard, A., Impact of Residual Modes in Structural Dynamics, *European Conference on Spacecraft Structures, Materials and Mechanical Testing, Noordwijk, The Netherlands, 10-12 May 2005.*